Quark Structure of Baryons: A 3D Harmonic Oscillator Model

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Introduction

Baryons (Ref. 9) are constructed of three spin-½ positive-parity quarks. There are six different quarks (Ref. 2):

- Up (u) quark: mass = (2.3 ± 0.6) MeV, charge = 2e/3, isospin = ½, i_z = 1/2
- Down (d) quark: mass = (4.8 ± 0.4) MeV, charge = -e/3, isospin = ½, i_z = -1/2
- Strange (s) quark: mass = (95 ± 5) MeV, charge = -e/3, strangeness = -1
- Charm (c) quark: mass = (1275 ± 25) MeV, charge = 2e/3, charm = 1
- Bottom (b) quark: mass = (4,180 ± 30) MeV, charge = -e/3, bottom = -1
- Top (t) quark: mass = (173,070 ± 720) MeV, charge =2e/3, top = 1

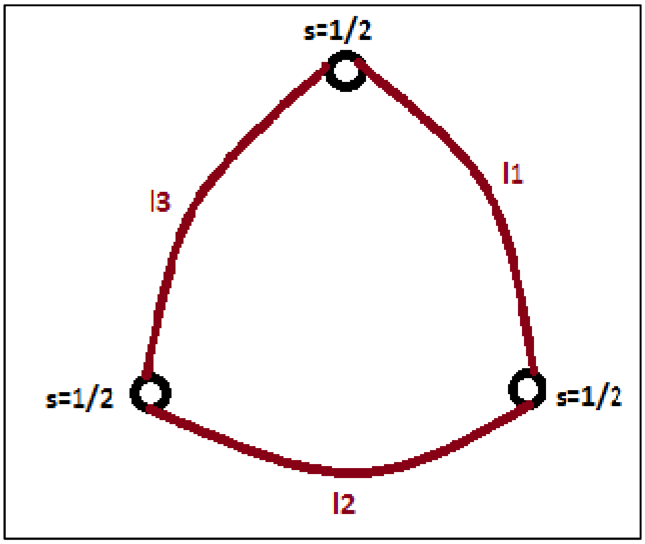
The quarks that compose different classes of baryons are:

- Proton: 2 u quarks and 1 d quark (uud)
- Neutron: 2 d quarks and 1 u quark (ddu)
- Delta baryons: uuu, uud, udd, ddd quark combinations (Ref. 10)
- Lambda baryons: uds, udc, udb, udt quark combinations (Ref. 11)
- Sigma baryons: uus, uds, dds, uuc, udc, ddc, uub, udb, ddb, uut, udt, ddt (Ref. 12)
- Xi baryons: uss, dss, uss, dss, usc, dsc, usc, dsc, ucc, dcc, usb, dsb, ubb, dbb, ucb, dcb (Ref. 13)
- Omega baryons: sss, ssc, ssb, scc, scb, sbb, ccc, ccb, cbb, bbb (Ref. 14)

This study only deals with the nucleons (proton and neutron), the delta barons, the lambda baryons and the Xi baryons.

Each of the three quarks in a baryon has a different color quantum number (red, blue, green) (Ref. 2). The color forces (Ref. 5) between each pair of quarks are such that the force increases with distance between the quarks; this is called “containment” (Ref. 3); however, at high energies the color force becomes weaker, called “asymptotic freedom” (Ref. 4). The color force field carriers are eight spin-1 gluons that carry color and anticolor quantum numbers (Ref. 4).

This a rough diagram of the color forces among the three quarks in a baryon:



Of course, the 3 quarks are not well localized particles; they would be better represented by a density cloud.

A simple force that increases with distance between the quarks is the 3D harmonic oscillator (3DHO) (Ref. 1). The study reported here applies three 3D harmonic oscillators to represent the containment color forces between the 3 quarks in a nucleon.

The 3D Harmonic Oscillator

The energy levels for the 3D harmonic oscillator are: $E = \hbar\omega\left(2n + \ell + \frac{3}{2}\right)$ for integer n and angular momentum ℓ (Ref. 1). The three quark masses (Ref. 11) are added to this equation. Thus, the energy equation for a baryon is $\boxed{E = \sum_{i=1}^{3} g_i\left(2n_i + \ell_i + \frac{3}{2}\right) + \sum_{i=1}^{3} m_i}$; the three quark masses are $\boxed{m_1, m_2, \mathrm{m}_3}$ and the three color-force (gluon) parameters to be varied in the fit to the resonance pole positions are $\boxed{g_1 = \hbar\omega_1 \ , g_2 = \hbar\omega_2 \ , g_3 = \hbar\omega_3}$.
They are the strengths of the three color forces binding the three quarks in the nucleon; they are not in a particular order with regard to the three quarks in the baryons. These three parameters are somewhat different for the different types of baryons.

Some general rules apply:

- The three sets of four quantum numbers $\left(n, \ell, m_\ell, m_s\right)$ for the three 3D oscillators must differ from each other set in at least one of the four quantum numbers according to the Pauli Principle (Ref. 8).
- The angular-momentum quantization direction is defined by one of the three quark ½ spins or one of the three quark angular momenta ℓ.
- The total angular momentum (j) must be 0 or positive.

The values of the 6 parameters ($n_1, \ell_1, n_2, \ell_2, n_3, \ell_3$) for the resonance-pole positions in the complex energy plane for a baryon or, when the pole positions are not known, the Breit-Wigner masses of a baryon resonance are determined to satisfy the conditions:

- $s_m = s_{1m} + s_{2m} + s_{3m} = \pm\frac{1}{2}$
- $\ell = \ell_{1m} + \ell_{2m} + \ell_{3m}$
- $j = \ell + s_m$
- Parity of a baryon is the sign of $(-1)^{l_1}(-1)^{l_2}(-1)^{l_3}$. Note that the sum $\ell_1 + \ell_2 + \ell_3$ has to be an odd number for an S ($\ell = 0$) state, even and >0 for a P ($\ell = 1$) state, odd and >1 for a D ($\ell = 2$)state, etc.
- The lowest-mass state of a baryon has the lowest possible set of the 6 parameters ($n_1, \ell_1, n_2, \ell_2, n_3, \ell_3$).
- The starting values of the (g_1, g_2, g_3) parameters are determined by requiring the calculated mass of the lowest state to be calculated exactly.

Fitting the Baryon Masses

This study fits the energy equation to the pole positions of the baryon resonances in the complex energy plane or the Breit-Wigner resonance parameters. There are many sets of 3DHO parameter sets that do not correspond to any known baryon resonances. Perhaps there are unknown baryon resonances and/or unknown selection rules besides the Pauli exclusion principal that exclud**e** some of those states.

The procedure for finding the three gluon parameters for a class of baryons is:

1. The lowest possible values of $(n_1, \ell_1, n_2, \ell_2, n_3, \ell_3)$ that yield the correct ℓ, j and + parity of the lowest state are selected. E.g., for the proton they are (1, 0, 0, 0, 0, 0). [Taking the lowest state as (0, 0, 0, 0, 0, 0) does not appear to give correct masses for the higher nucleon states.)
2. The three gluon parameters, (g_1, g_2, g_3), are set to 1 and then varied to fit the measured mass of the lowest state.
3. A table is made of the $2^6 = 64$ values 0 or 1 for the $(n_1, \ell_1, n_2, \ell_2, n_3, \ell_3)$ along with the calculated parity and masses using the three gluon parameters determined in step 2. See the Appendix for a sample table.
4. The lowest calculated masses and correct parity for the first and second higher states are selected from the table of step 3 and the three gluon parameters are varied to fit the lowest three states.
5. The three gluon parameters determined in step 4 are used to recalculate the table of step 3. Then the closest masses with proper parity for higher states are selected.
6. Some higher states cannot be represented by only 0 and 1 values for the four gluon parameters. For those states guessed higher values have to be tried. (A table of all 0 or 1 or 2 values would have $3^6 = 729$ rows.)
7. Then the three gluon parameters are varied again to fit the masses of all of the states.

Only baryon types are considered for which there are at least three known members with well-established relevant parameters.

Nucleon Resonances

The nucleon resonances are designated by the symbols:

Symbol	ℓ	j	Parity
S_{11}, S_{31}	0	$\frac{1}{2}$	−
P_{11}, P_{31}	1	$\frac{1}{2}$	+
P_{13}, P_{33}	1	$\frac{3}{2}$	+
D_{13}, D_{33}	2	$\frac{3}{2}$	−
D_{15}, D_{35}	2	$\frac{5}{2}$	−
F_{15}, F_{35}	3	$\frac{5}{2}$	+
F_{17}, F_{37}	3	$\frac{7}{2}$	+
G_{17}, G_{37}	4	$\frac{7}{2}$	−
G_{19}, G_{39}	4	$\frac{9}{2}$	−
H_{19}, H_{39}	5	$\frac{9}{2}$	+
$H_{1,11}, H_{3,11}$	5	$\frac{11}{2}$	+

The first index specifies the isospin, either ½ or $\frac{3}{2}$ without the denominator 2. The second index specifies the total angular momentum j without the denominator 2, which is listed in the 3rd column. ℓ is the orbital angular momentum. The parity is the sign of $-(-1)^{\ell}$, because the pion in the beam that produces the resonance when colliding with the nucleon has negative parity (Ref. 7).

For the isospin-½ resonances the + electric charge state is used, which contains 2 u quarks and 1 d quark. For the isospin-$\frac{3}{2}$ resonances the 2+ electric charge state is used, which contains 3 u quarks.

This study uses the pole positions of the nucleon resonances in the complex energy plane. There are two sources for the resonance pole positions: the SAID project at George Washington University (Ref. 15) and the Particle Data Tables (Ref. 16).

Fitting the Nucleon Pole Positions

Fits for Isospin ½ Nucleon Resonances

These are the results for the isospin-½ nucleons. The first table lists the four parameters for the d quark and one of the u quarks; the second table lists the four parameters for the second u quark, the four parameters for the resonance and the calculated and measured mass for the nucleon (Ref. 15).

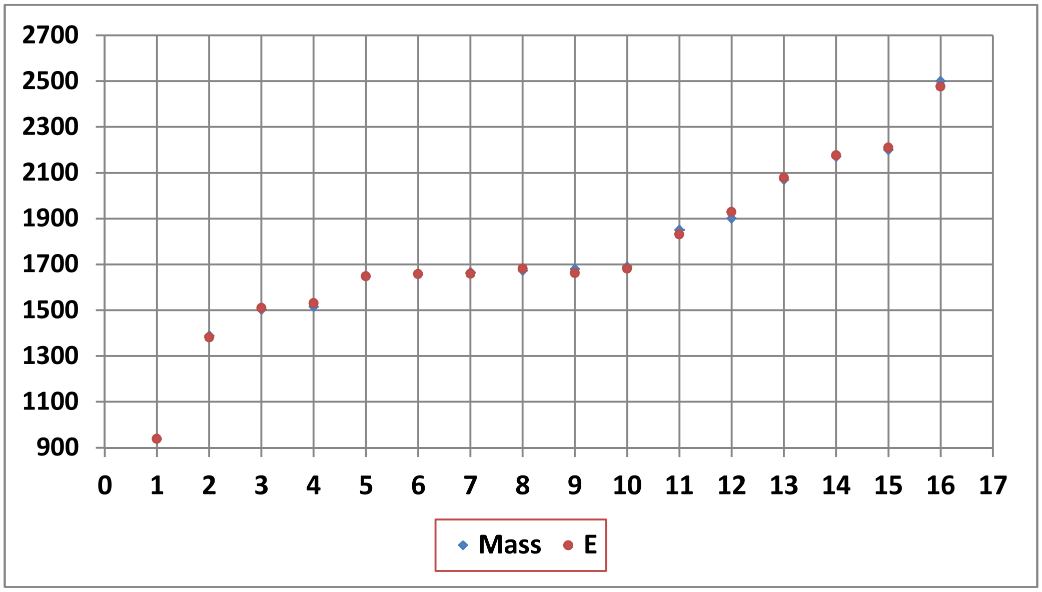
						Charge:	+2/3		-1/3		
g1:	**150.04**	**g2:**	**236.56**	**g3:**	**32.59**	**u mass:**	**2.30**	**d mass:**	**4.80**		
Q1n	**Q1l**	Q1ml	Q1ms	Q2n	**Q2l**	Q2ml	Q2ms	Q3n	**Q3l**	Q3ml	Q3ms
1	**0**	**0**	**1/2**	**0**	**0**	**0**	**1/2**	**0**	**0**	**0**	**- 1/2**
0	**0**	**0**	**1/2**	**1**	**1**	**0**	**- 1/2**	**0**	**1**	**1**	**- 1/2**
1	**0**	**0**	**1/2**	**1**	**0**	**0**	**- 1/2**	**1**	**1**	**0**	**1/2**
0	**1**	**0**	**- 1/2**	**1**	**1**	**1**	**- 1/2**	**0**	**1**	**1**	**1/2**
1	**0**	**0**	**1/2**	**1**	**1**	**0**	**1/2**	**0**	**0**	**0**	**- 1/2**
2	**1**	**0**	**1/2**	**0**	**1**	**1**	**- 1/2**	**0**	**1**	**1**	**1/2**
1	**1**	**0**	**1/2**	**1**	**0**	**0**	**1/2**	**1**	**1**	**1**	**- 1/2**
0	**2**	**1**	**- 1/2**	**1**	**1**	**1**	**- 1/2**	**0**	**1**	**1**	**1/2**
0	**1**	**0**	**- 1/2**	**1**	**1**	**1**	**- 1/2**	**2**	**1**	**1**	**1/2**
1	**0**	**0**	**1/2**	**1**	**1**	**0**	**- 1/2**	**0**	**1**	**1**	**- 1/2**
1	**1**	**0**	**- 1/2**	**1**	**1**	**1**	**- 1/2**	**0**	**1**	**1**	**1/2**
1	**1**	**1**	**1/2**	**1**	**1**	**-1**	**1/2**	**1**	**2**	**1**	**- 1/2**
1	**2**	**1**	**- 1/2**	**1**	**1**	**1**	**1/2**	**1**	**2**	**2**	**- 1/2**
1	**2**	**2**	**- 1/2**	**1**	**1**	**1**	**1/2**	**2**	**3**	**2**	**- 1/2**
1	**2**	**1**	**- 1/2**	**1**	**1**	**2**	**1/2**	**3**	**2**	**1**	**1/2**
2	**2**	**2**	**1/2**	**1**	**1**	**1**	**1/2**	**2**	**3**	**2**	**- 1/2**

lm	**sm**	**j**	**parity**	**E**	**Mass**	State		Diff
0	**1/2**	**1/2**	**+**	**938**	**938.3**	p	1	0
1	**- 1/2**	**1/2**	**+**	**1380**	**1388**	P11	2	7.53
0	**1/2**	**1/2**	**-**	**1509**	**1502**	S11	3	-7.17
2	**- 1/2**	**1 1/2**	**-**	**1531**	**1515**	D13	4	-15.51
0	**1/2**	**1/2**	**-**	**1648**	**1648**	S11	5	0.03
2	**1/2**	**2 1/2**	**-**	**1658**	**1656**	D15	6	-1.55
1	**1/2**	**1 1/2**	**+**	**1659**	**1665**	P13	7	5.79
3	**- 1/2**	**2 1/2**	**+**	**1681**	**1673**	F15	8	-7.55
2	**- 1/2**	**1 1/2**	**-**	**1661**	**1680**	D13	9	19.14
1	**- 1/2**	**1/2**	**+**	**1681**	**1690**	P11	10	9.45
2	**- 1/2**	**1 1/2**	**-**	**1831**	**1850**	D13	11	19.40
1	**1/2**	**1 1/2**	**+**	**1928**	**1900**	P13	12	-28.36
4	**- 1/2**	**3 1/2**	**-**	**2078**	**2070**	G17	13	-8.40
5	**- 1/2**	**4 1/2**	**+**	**2176**	**2170**	H19	14	-6.16
4	**1/2**	**4 1/2**	**-**	**2209**	**2200**	G19	15	-8.75
5	**1/2**	**5 1/2**	**+**	**2476**	**2500**	H1,11	16	23.75
							Sum	1.64

The equation for calculating the masses is $\boxed{E=\sum_{i=1}^{3} g_i\left(2n_i+\ell_i+\tfrac{3}{2}\right)+2(2.3)+4.8}$. These calculated masses are well within the uncertainties in the measured masses.

The fit was restricted to exactly produce the measured proton mass as the three parameters were varied to produce as closely as fit as possible to the remaining isospin-½ nucleon masses.

The graph below shows how closely the calculated energies correspond to the measured nucleon masses:



Fits for Isospin 3/2 Nucleon Resonances

These are the results for the isospin-$\frac{3}{2}$ nucleons. The first table lists the four parameters for the first u quark and the second u quark and the third u quark. The second table lists the four parameters for the resonance and the calculated and measured mass for the resonance (Refs. 10 & 15).

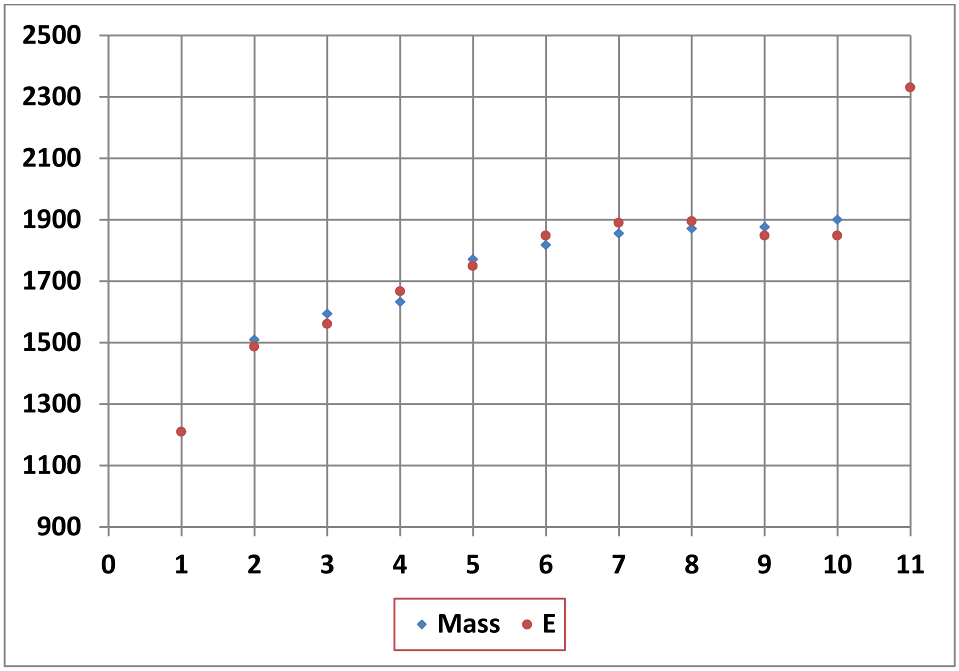
g1:	286.92	g2:	187.85	g3:	81.61	u mass:	2.3				
Q1n	Q1l	Q1ml	Q1ms	Q2n	Q2l	Q2ml	Q2ms	Q3n	Q3l	Q3ml	Q3ms
0	1	0	1/2	0	0	0	1/2	0	1	1	- 1/2
0	0	0	1/2	1	1	0	1/2	0	1	1	- 1/2
0	1	-1	1/2	0	1	0	1/2	1	1	1	- 1/2
0	1	1	1/2	1	0	0	- 1/2	0	2	1	- 1/2
0	1	0	1/2	1	0	0	- 1/2	1	1	1	- 1/2
1	0	0	1/2	0	1	1	- 1/2	1	1	2	- 1/2
1	1	0	1/2	0	1	1	- 1/2	0	0	0	- 1/2
0	0	0	1/2	1	1	1	- 1/2	2	2	1	1/2
0	2	1	- 1/2	0	1	1	1/2	1	1	1	1/2
1	0	0	1/2	0	1	0	1/2	1	1	1	- 1/2
0	2	2	1/2	1	2	1	1/2	0	2	2	- 1/2

lm	sm	j	parity	E	Mass			Diff
1	1/2	1 1/2	+	1210	1210	P33	1	0
1	1/2	1 1/2	+	1487	1510	P33	2	23.38
0	1/2	1/2	-	1561	1594	S31	3	32.93
2	- 1/2	1 1/2	-	1667	1632	D33	4	-35.31
1	- 1/2	1/2	+	1749	1771	P31	5	22.08
3	- 1/2	2 1/2	+	1848	1818	F35	6	-29.99
1	- 1/2	1/2	+	1890	1855	P31	7	-35.07
2	1/2	2 1/2	-	1895	1871	D35	8	-23.69
3	1/2	3 1/2	+	1848	1876	F37	9	28.01
1	1/2	1 1/2	+	1848	1900	P33	10	52.01
5	1/2	5 1/2	+	2330	2330	H3,11	11	0.08
							Sum	34.41

The equation for calculating the masses is $\boxed{E=\sum_{i=1}^{3} g_i\left(2n_i+\ell_i+\frac{3}{2}\right)+3(2.3)}$. These calculated masses are well within the uncertainties in the measured masses.

The fit was required to exactly produce the measured Δ_{33} (P_{33}) mass as the three parameters were varied to produce a fit as closely as possible to the remaining isospin-3/2 nucleon masses.

The graph below shows how closely the calculated energies correspond to the measured nucleon masses:



Fits for Lambda (Λ) Baryons

These are the results for the Lambda (Λ) baryons. The first table lists the four parameters for the first u quark and the second d quark and the third s quark. The second table lists the four parameters for the baryon and the calculated and measured mass for the baryon (Ref. 11).

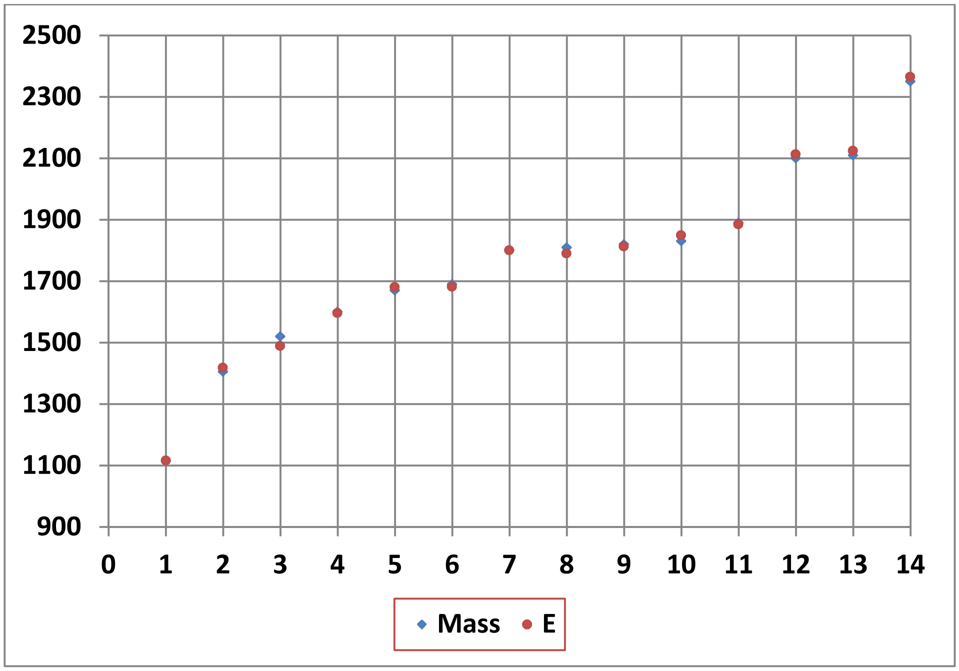
g1:	155.63	g2:	204.50	g3:	108.08	u mass:	2.30	d mass:	4.80	s mass:	95
Q1n	Q1l	Q1ml	Q1ms	Q2n	Q2l	Q2ml	Q2ms	Q3n	Q3l	Q3ml	Q3ms
1	0	0	1/2	0	0	0	1/2	0	0	0	- 1/2
0	0	0	1/2	1	1	0	1/2	0	0	0	- 1/2
0	1	0	1/2	0	1	1	- 1/2	1	1	1	- 1/2
1	1	0	- 1/2	0	0	0	- 1/2	1	1	1	1/2
1	1	0	1/2	1	0	0	1/2	0	0	0	- 1/2
0	1	0	1/2	1	1	1	- 1/2	0	1	1	- 1/2
1	1	0	1/2	0	1	0	1/2	1	1	0	- 1/2
0	1	0	- 1/2	1	1	1	- 1/2	1	0	0	1/2
0	1	1	- 1/2	0	1	1	1/2	2	2	1	- 1/2
1	0	0	1/2	1	0	0	1/2	0	3	2	- 1/2
1	1	0	- 1/2	1	1	1	1/2	0	0	0	1/2
1	1	1	1/2	0	2	1	- 1/2	1	2	2	- 1/2
1	1	1	- 1/2	0	1	1	1/2	2	2	1	- 1/2
1	2	2	- 1/2	1	1	1	1/2	0	3	2	1/2

lm	sm	j	parity	E	Mass	State		Diff
0	1/2	1/2	+	1116	1116	Lambda	1	0
0	1/2	1/2	-	1418	1405	S01	2	-12.84
2	- 1/2	1 1/2	-	1489	1519.5	D03	3	30.69
1	- 1/2	1/2	+	1596	1600	P01	4	4.43
0	1/2	1/2	-	1680	1670	S01	5	-10.32
2	- 1/2	1 1/2	-	1682	1690	D03	6	8.35
0	1/2	1/2	-	1800	1800	S01	7	-0.07
1	- 1/2	1/2	+	1790	1810	P01	8	20.26
3	- 1/2	2 1/2	+	1813	1820	F05	9	6.93
2	1/2	2 1/2	-	1849	1830	D05	10	-18.94
1	1/2	1 1/2	+	1885	1890	P03	11	5.18
4	- 1/2	3 1/2	-	2113	2100	G07	12	-12.66
3	- 1/2	2 1/2	+	2124	2110	F05	13	-14.32
5	1/2	5 1/2	+	2365	2350	H1,11	14	-14.71
							Sum:	-8.01

The equation for calculating the masses is $\boxed{E = \sum_{i=1}^{3} g_i \left(2n_i + \ell_i + \tfrac{3}{2}\right) + 2.3 + 4.8 + 95}$. These calculated masses are well within the uncertainties in the measured masses.

The fit was required to exactly produce the measured Λ mass as the three parameters were varied to produce a fit as closely as possible to the remaining Λ-baryon masses.

The graph below shows how closely the calculated energies correspond to the measured Λ-baryon masses:



Fits for Sigma (Σ) Baryons

These are the results for the Sigma (Σ) baryons. The first table lists the four parameters for the first u quark and the second u quark and the third s quark. The second table lists the four parameters for the baryon and the calculated and measured mass for the baryon (Ref. 12).

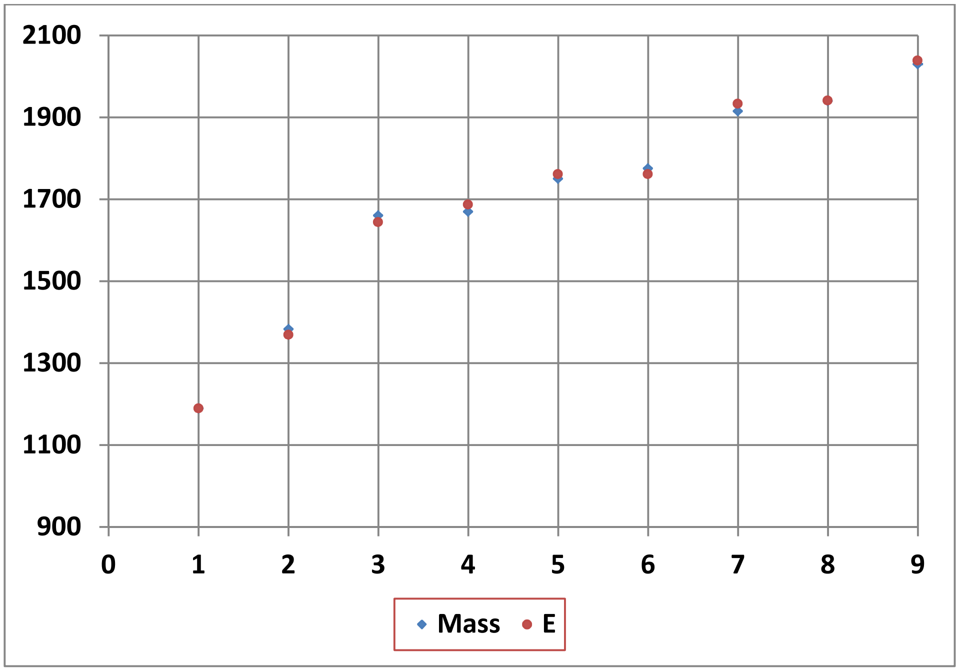
g1:	171.71	g2:	208.81	g3:	117.04	u mass:	2.30	s mass:	95		
Q1n	Q1l	Q1ml	Q1ms	Q2n	Q2l	Q2ml	Q2ms	Q3n	Q3l	Q3ml	Q3ms
1	0	0	1/2	0	0	0	1/2	0	0	0	- 1/2
0	1	0	1/2	0	0	0	1/2	1	1	1	- 1/2
0	1	0	1/2	1	1	1	- 1/2	0	0	0	- 1/2
1	1	1	1/2	0	1	1	- 1/2	0	1	0	- 1/2
0	1	0	1/2	1	1	0	1/2	0	1	0	- 1/2
0	1	0	1/2	1	1	1	- 1/2	0	1	1	1/2
0	2	1	1/2	1	1	1	- 1/2	0	1	1	- 1/2
0	0	0	- 1/2	1	1	1	- 1/2	1	2	1	1/2
1	1	1	- 1/2	0	1	1	- 1/2	1	2	1	1/2

lm	sm	j	parity	E	Mass	State		Diff
0	1/2	1/2	+	1189	1189	$\Sigma+$	1	0
1	1/2	1 1/2	+	1369	1383	P13	2	14.03
1	- 1/2	1/2	+	1644	1660	P11	3	15.90
2	- 1/2	1 1/2	-	1687	1670	D13	4	-16.93
0	1/2	1/2	-	1761	1750	S11	5	-11.14
2	1/2	2 1/2	-	1761	1775	D15	6	13.86
3	- 1/2	2 1/2	+	1933	1915	F15	7	-17.85
2	- 1/2	1 1/2	-	1941	1940	D13	8	-0.53
3	- 1/2	2 1/2	+	2038	2030	F17	9	-8.04
							Sum:	-10.70

The equation for calculating the masses is $\boxed{E=\sum_{i=1}^{3} g_i\left(2n_i+\ell_i+\tfrac{3}{2}\right)+2(2.3)+95}$.These calculated masses are well within the uncertainties in the measured masses.

The fit was required to exactly produce the measured Σ mass as the three parameters were varied to produce a fit as closely as possible to the remaining Σ-baryon masses.

The graph below shows how closely the calculated energies correspond to the measured Σ-baryon masses:



Fits for Xi (Ξ) Baryons

These are the results for the Xi (Ξ) baryons. The first table lists the four parameters for the first u quark and the second s quark and the third s quark. The second table lists the four parameters for the baryon and the calculated and measured mass for the baryon (Ref. 13).

g1:	189.74	g2:	171.45	g3:	137.11	d mass:	4.80	s mass:	95		
Q1n	Q1l	Q1ml	Q1ms	Q2n	Q2l	Q2ml	Q2ms	Q3n	Q3l	Q3ml	Q3ms
1	0	0	1/2	0	0	0	1/2	0	0	0	- 1/2
0	0	0	1/2	0	1	1	1/2	1	1	0	- 1/2
1	1	0	1/2	0	1	1	- 1/2	0	1	1	- 1/2

lm	sm	j	parity	E	Mass	State		Diff
0	1/2	1/2	+	1322	1322	Xi-	1	0
1	1/2	1 1/2	+	1525	1525	P13	2	0.00
2	- 1/2	1 1/2	-	1820	1820	D13	3	0.00
							Sum:	0.00

The equation for calculating the masses is $\boxed{E = \sum_{i=1}^{3} g_i \left(2n_i + \ell_i + \tfrac{3}{2}\right) + 2.3 + 2(95)}$. These calculated masses are exact because there are three parameters to fit three data.

Fits for Lambda-Charm (Λ_c) Baryons

These are the results for the Lambda-Charm (Λ_c) baryons. The first table lists the four parameters for the first u quark and the second d quark and the third c quark. The second table lists the four parameters for the baryon and the calculated and measured mass for the baryon (Ref. 11).

g1:	156.94	g2:	206.31	g3:	97.06	u mass:	2.30	d mass:	4.80	c mass:	1275
Q1n	Q1l	Q1ml	Q1ms	Q2n	Q2l	Q2ml	Q2ms	Q3n	Q3l	Q3ml	Q3ms
1	0	0	1/2	0	0	0	1/2	0	0	0	- 1/2
0	0	0	1/2	1	1	0	1/2	0	0	0	- 1/2
0	1	0	1/2	0	1	1	- 1/2	1	1	1	- 1/2
0	0	0	1/2	1	1	1	- 1/2	0	3	2	- 1/2

lm	sm	j	parity	E	Mass	State		Diff
0	1/2	1/2	+	2286	2286	LambdaC	1	0
0	1/2	1/2	-	2592	2592	S01	2	0.74
2	- 1/2	1 1/2	-	2627	2628	D03	3	1.11
3	- 1/2	2 1/2	+	2883	2882	F05	4	-1.15
							Sum:	0.70

Fits for Xi-Charm (Ξ_c) Baryons

These are the results for the Xi (Ξ_c) baryons. The first table lists the four parameters for the first u quark and the second u quark and the third c quark. The second table lists the four parameters for the baryon and the calculated and measured mass for the baryon (Ref. 13).

						Charge:	2/3		-1/3		2/3
g1:	191.27	g2:	143.23	g3:	139.13	d mass:	4.80	s mass:	95	c mass:	1275
Q1n	Q1l	Q1ml	Q1ms	Q2n	Q2l	Q2ml	Q2ms	Q3n	Q3l	Q3ml	Q3ms
1	0	0	1/2	0	0	0	1/2	0	0	0	- 1/2
0	0	0	1/2	0	1	1	1/2	1	1	0	- 1/2
0	0	0	1/2	1	0	0	- 1/2	1	1	0	- 1/2
0	1	0	1/2	0	1	1	- 1/2	1	1	1	- 1/2

lm	sm	j	parity	E	Mass	State		Diff
0	1/2	1/2	+	2467.8	2468	XiC-	1	0
1	1/2	1 1/2	+	2645.88	2646	P13	2	0.02
0	- 1/2	- 1/2	-	2789.1	2789	S11	3	0.00
2	- 1/2	1 1/2	-	2837.15	2817	D13	3	-20.55
							Sum:	-20.53

The equation for calculating the masses is $\boxed{E=\sum_{i=1}^{3} g_i\left(2n_i+\ell_i+\frac{3}{2}\right)+2(2.3)+1275}$.

Fits for Lambda-Bottom (Λ_b) Baryons

These are the results for the Lambda-Charm (Λ_b) baryons. The first table lists the four parameters for the first u quark and the second d quark and the third b quark. The second table lists the four parameters for the baryon and the calculated and measured mass for the baryon (Ref. 11).

						Charge:	+2/3		-1/3		+2/3
g1:	253.84	g2:	266.76	g3:	95.83	u mass:	2.30	d mass:	4.80	c mass:	4180
Q1n	Q1l	Q1ml	Q1ms	Q2n	Q2l	Q2ml	Q2ms	Q3n	Q3l	Q3ml	Q3ms
1	0	0	1/2	0	0	0	1/2	0	0	0	- 1/2
0	0	0	1/2	1	1	0	1/2	0	0	0	- 1/2
0	1	0	1/2	0	1	1	- 1/2	1	1	1	- 1/2

lm	sm	j	parity	E	Mass	State		Diff
0	1/2	1/2	+	5619	5619	LambdaB	1	0
0	1/2	1/2	-	5912	5912	S01	2	0.00
2	- 1/2	1 1/2	-	5920	5920	D03	3	0.00
							Sum:	0.00

The equation for calculating the masses is $\boxed{E=\sum_{i=1}^{3} g_i\left(2n_i+\ell_i+\frac{3}{2}\right)+2.3+4.8+4180}$. These calculated masses are exact because there are three parameters to fit three data.

Conclusions

This study shows that the baryon resonances can be accurately matched by quantum states of three 3D harmonic oscillators (3DHO) representing the three color forces that bind the three quarks that compose the baryon resonances. There are many 3DHO states that give masses different than known baryon resonances.

Previous attempts to use the 3DHO model for baryons (Ref. 20) used only one 3DHO for all three quarks. It did not work because the S11 state had a lower mass than the P11 (Roper) state, with is wrong.

Using three separate 3DHOs for the three quark interactions in this analysis fixes that problem. Here are the quantum numbers and calculated parameters for the first three nucleon states:

Q1n	Q1l	Q1ml	Q1ms	Q2n	Q2l	Q2ml	Q2ms	Q3n	Q3l	Q3ml	Q3ms
1	0	0	1/2	0	0	0	1/2	0	0	0	- 1/2
0	0	0	1/2	1	1	0	- 1/2	0	1	1	- 1/2
1	0	0	1/2	1	0	0	- 1/2	1	1	0	1/2

lm	sm	j	parity	E	Mass	State
0	1/2	1/2	+	938	938.3	p
1	- 1/2	1/2	+	1380	1388	P11
0	1/2	1/2	-	1509	1502	S11

The 3DHO model is non-relativistic, but it appears to work quite well. There is a well-known lowest-order relativistic correction for the 3DHO (Ref. 19), but it does not seem to be necessary for calculating the nucleon masses using the 3DHO model.

The three $\boxed{g_1 = \hbar\omega_1 \ , g_2 = \hbar\omega_2 \ , g_3 = \hbar\omega_3}$ parameters are summarized for the eight baryon types in this study:

Color-Force parameters

Baryon:	G1:	G2:	G3:
N1/2:	150.04	236.56	32.59
N3/2:	286.92	187.85	81.61
Lambda:	155.63	204.50	108.08
Sigma:	171.71	208.81	117.04
Xi:	189.74	171.45	137.11
LambdaC:	156.94	206.31	97.06
XiC:	191.27	143.23	139.13
LambdaB:	253.84	266.76	95.83

They are not in a particular order with regard to the three quarks in the baryons.

The unsolved aspects of this analysis are:

- Why there are many states that are not represented in the data?
- Why do the masses come out close to measured values for a nonrelativistic 3DHO model?

References

1. http://en.wikipedia.org/wiki/Quantum_harmonic_oscillator#Example:_3D_isotropic_harmonic_oscillator
2. http://en.wikipedia.org/wiki/Quark
3. http://en.wikipedia.org/wiki/Quark_confinement
4. http://en.wikipedia.org/wiki/Gluon
5. The Color Force
6. http://en.wikipedia.org/wiki/Asymptotic_freedom
7. http://en.wikipedia.org/wiki/Pion
8. http://en.wikipedia.org/wiki/Pauli_principle
9. http://en.wikipedia.org/wiki/Baryons
10. http://en.wikipedia.org/wiki/Delta_baryon
11. http://en.wikipedia.org/wiki/Lambda_baryon
12. http://en.wikipedia.org/wiki/Sigma_baryon
13. http://en.wikipedia.org/wiki/Xi_baryon
14. http://en.wikipedia.org/wiki/Omega_baryon
15. http://gwdac.phys.gwu.edu/analysis/pin_analysis.html
16. http://pdg.lbl.gov/2013/tables/rpp2013-sum-baryons.pdf
17. http://pdg.lbl.gov/2013/tables/rpp2013-sum-quarks.pdf
18. http://www.phy.duke.edu/~kolena/modern/hansen.html
19. http://physicspages.com/2013/08/15/harmonic-oscillator-relativistic-correction/
20. On the nature of the Roper resonance

27 January 2014
http://arts.bev.net/roperldavid/

Appendix: Table for sixty-four 0 and 1 values for Nucleon 3DHO Parameters

u mass:	2.30	d mass:	4.80						
g1:	150.04	g2:	236.56	g3:	32.59				
dn	dl	u1n	u1l	u2n	u2l	parity	E		
0	0	0	0	0	0	+	638.2		1
0	0	0	0	0	1	-	670.8		2
0	0	0	0	1	0	+	703.4		3
0	0	0	0	1	1	-	736		4
0	1	0	0	0	0	-	788.2		5
0	1	0	0	0	1	+	820.8		6
0	1	0	0	1	0	-	853.4		7
0	0	0	1	0	0	-	874.8		8
0	1	0	0	1	1	+	886		9
0	0	0	1	0	1	+	907.3		10
1	0	0	0	0	0	+	938.3	proton	11
0	0	0	1	1	0	-	939.9		12
1	0	0	0	0	1	-	970.9		13
0	0	0	1	1	1	+	972.5		14
1	0	0	0	1	0	+	1003		15
0	1	0	1	0	0	+	1025		16
1	0	0	0	1	1	-	1036		17
0	1	0	1	0	1	-	1057		18
1	1	0	0	0	0	-	1088		19
0	1	0	1	1	0	+	1090		20
0	0	1	0	0	0	+	1111		21
1	1	0	0	0	1	+	1121		22
0	1	0	1	1	1	-	1123		23
0	0	1	0	0	1	-	1144		24
1	1	0	0	1	0	-	1153		25
1	0	0	1	0	0	-	1175		26
0	0	1	0	1	0	+	1176		27
1	1	0	0	1	1	+	1186		28
1	0	0	1	0	1	+	1207		29
0	0	1	0	1	1	-	1209		30
1	0	0	1	1	0	-	1240		31
0	1	1	0	0	0	-	1261		32
1	0	0	1	1	1	+	1273		33
0	1	1	0	0	1	+	1294		34
1	1	0	1	0	0	+	1325		35
0	1	1	0	1	0	-	1327		36
0	0	1	1	0	0	-	1348		37
1	1	0	1	0	1	-	1357		38
0	1	1	0	1	1	+	1359		39

0	0	1	1	0	1	+	1380	P11	40
1	1	0	1	1	0	+	1390		41
1	0	1	0	0	0	+	1411		42
0	0	1	1	1	0	-	1413		43
1	1	0	1	1	1	-	1423		44
1	0	1	0	0	1	-	1444		45
0	0	1	1	1	1	+	1446		46
1	0	1	0	1	0	+	1477		47
0	1	1	1	0	0	+	1498		48
1	0	1	0	1	1	-	1509	S11	49
0	1	1	1	0	1	-	1531	D13	50
1	1	1	0	0	0	-	1561		51
0	1	1	1	1	0	+	1563		52
1	1	1	0	0	1	+	1594		53
0	1	1	1	1	1	-	1596		54
1	1	1	0	1	0	-	1627		55
1	0	1	1	0	0	-	1648	S11	56
1	1	1	0	1	1	+	1659	P13	57
1	0	1	1	0	1	+	1681	P11	58
1	0	1	1	1	0	-	1713		59
1	0	1	1	1	1	+	1746		60
1	1	1	1	0	0	+	1798		61
1	1	1	1	0	1	-	1831	D13	62
1	1	1	1	1	0	+	1863		63
1	1	1	1	1	1	-	1896		64