Fundamental-Particles Model for Black Holes

L. David Roper, roperld@vt.edu, http://roperld.com/personal/RoperLDavid.htm

16 April 2019

Introduction

The most realistic model for the exterior of a black hole (BH) is the Kerr solution to Einstein's General Relativity equation. That model is for a black hole with spin and all cosmic black holes have spin. The interior of the Kerr Model is delineated by the event horizon (EH), the inside of which an external observer cannot make measurements, since no object, including photons, can escape after it goes inside the event horizon. Thus the Kerr Model's predictions of what happens inside the EH of a black hole cannot be proven by observations. (For the unrealistic case of zero spin BHs the Kerr Model becomes the much simpler Schwarzschild Model.)

The model for the interior of a spinning black hole described in this article has the following properties:

1. Matter than enters the event horizon of a spinning black hole is eventually deconstructed into the stable fundamental particles of the Standard Model of particle physics, which are assumed to be unable to be broken into more fundamental units, even by very strong gravity.
2. The ring singularity of the Kerr Model of a black hole is replaced by a very thin rotating ring torus, which is very thin but not a singularity.
3. The ring torus contains the stable fundamental particles of the Standard Model of particle physics: quarks, electrons and perhaps electron neutrinos and photons.
4. The fundamental particles are cubes with Planck-Length sides packed together in the ring torus.
5. The rotation of the ring torus provides the spin of the black hole. (The spins of the remnant black holes detected by LIGO are all close to $J = 0.7\frac{GM^2}{c}$. See Appendix below.)

Background

The external gravitational field beyond the event horizon of a BH only depends on a spherically symmetric interior for the Schwarzschild Mode, not some specific internal properties of the symmetric sphere. For a spinning black hole according to the Kerr Model the event horizon is a symmetric oblate ellipsoid instead of a sphere, with the spin-vector direction along the minor axis.

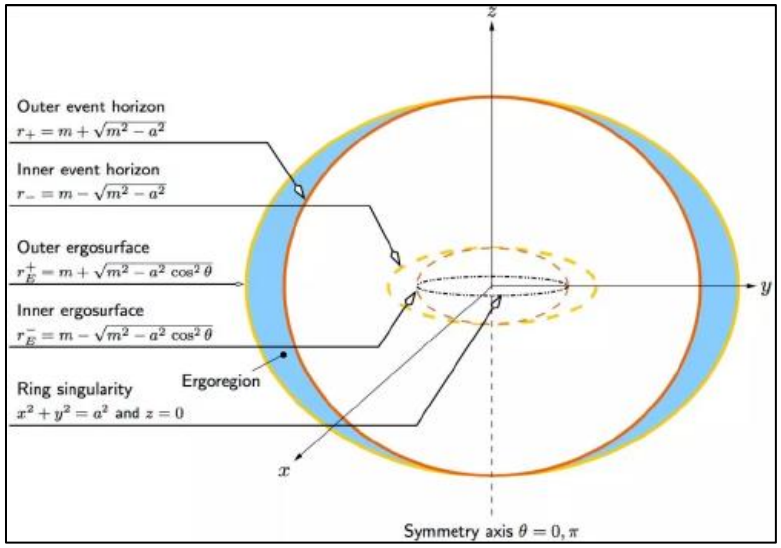
The Schwarzschild Model of a BH has all the mass in an unphysical singularity at the center. The Kerr Model has specific mathematics of what is inside the Kerr event horizon (EH), including a ring singularity that contains all the mass:

$$x^2 + y^2 = a^2 \text{ and } z = 0, \text{where } a \equiv \frac{J}{cM} \text{ for spin } J, \text{ speed of light } c \text{ and black-hole mass } M.$$

x and y are in the spin plane at the center of the black hole and z is the spin direction.

All of the BH mass M is at radius a, a ring singularity.

The mechanical definition of spin for a torus of radius a is $J = Mva$, but if $J = acM$ then $v = c$, a violation of the $v < c$ rule of special relativity. This is another reason to not accept the Kerr Model inside the event horizon.



https://www.markushanke.net/the-kerr-spacetime/

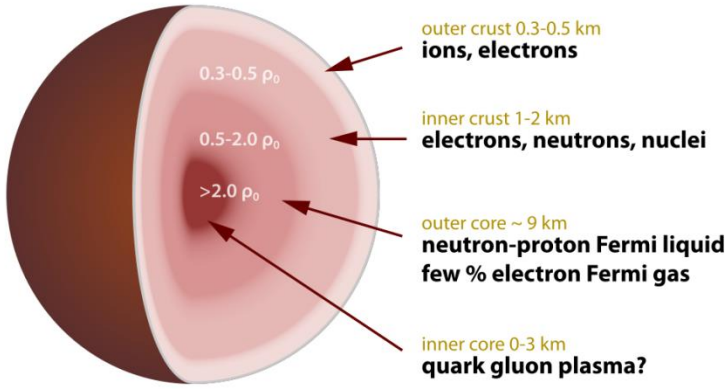
This article argues against an unphysical ring singularity centered at the center of the BH. It is argued that, instead of a ring singularity, there is a ring torus at the location of the Kerr ring singularity which contains the stable fundamental particles of the Standard Model each in a quantized cube of Planck-Length size. The tube of the ring torus, located with its tube center at the a radius given above for a ring singularity, is very small even for very large BHs. So, it simulates a ring singularity.

Quantum theory does not allow space singularities; instead at very small distances quantization of space must surely occur; i.e. gravity must be quantized. The model of the inside of an event horizon of a black hole describes here assumes that, at very small distances, space is quantized by the dimensions of the Planck Length. (See the appendix about Planck Units.) So, in this model the ring singularity is replaced by a very small ring torus in the spin plane at the center of the black hole. (See an appendix for ring torus equations.)

One could devise many models for what occurs inside the EH of a black hole. This article discusses one possible internal model for the interior of a BH that is based on known physics of the Standard Model of particle physics and the Planck Length. (See the appendix for the fundamental particles of the Standard Model.) The **stable fundamental particles** of the Standard Model are the u and d quarks of three "colors" each, eight zero-mass gluons, the electron, the electron-neutrino and the zero-mass photon. The model of the inside of the event horizon of a black hole described here assumes that the stable fundamental particles of the Standard Model are indestructible even in the strong gravity of a BH and occupy the space of a ring torus with dimensions determined by the Planck Length and the mass M. The strong gravity of the BH is causing the stable fundamental particles to pack as close as possible, so gluons that hold the quarks together in nucleons disappear when the fundamental particles enter the ring torus.

The Kerr Model predicts that near or inside the EH the gravitational tidal forces are so strong that objects that go there are ripped apart (called "spaghettification") and are quickly transported to a ring singularity around the center in the spin plane. For small black holes that ripping apart may start before crossing the EH and for very large (supermassive) blacks holes it may start well inside the EH, because the EH radius is proportional to the BH mass and the ripping force at the EH is inversely proportional to the inverse of the BH mass ($F \approx M/r_{EH}^2 \approx 1/M$

Black holes are the ultimate compact large objects in the universe; the next most compact large objects are neutron stars which are made of spherical layers of fermions (protons, electrons, neutrons and, perhaps, quarks in the center):



Since quarks are much smaller and less massive than neutrons, it is possible that the more compact black holes are filled with quarks, electrons, electron-neutrinos and photons, which, according to current knowledge, are the fundamental particles out of which all matter is constructed. If the non-reducible constituents of all matter are quarks, electrons, electron-neutrinos and photons, spaghettification of objects upon entering a black hole should deconstruct the objects into those constituents.

The model describe here for the interior of a BH is just a mental exercise using known physics with no hope of verification, since outside observers cannot make measurements inside the event horizon

A Fundamental-Particles Model for Black Holes

Assumptions:

1. Black holes (BH) have zero net electric charge and nonzero spin, so their exterior properties can be represented by the Kerr Model solution of General Relativity. There is some observational evidence of this. While this article was being written the Event Horizon Telescope reported the first image of a the M87 supermassive black hole with mass 6.5 x 10^9 $M_\odot$ and 55 x 10^6 light years from Earth.
2. Since matter, including photons, that pass through the event horizon (EH) of a BH cannot exit the EH, there is no assurance that the Kerr Model represents the interior of a BH.
3. Matter plasma particles spiral inside the event horizon from the highly excited rotating accretion disk around the equator of the BH. The ions, electrons, electron neutrinos and photons spiral on toward the ring torus at the center. Their entering kinetic energy and angular momentum goes into the rotation of the ring torus at speeds near the speed of light, depending on the spin.
4. Ions that enter black holes are deconstructed into their constituent nucleons and electrons by the increasingly very strong gravity tidal force ("spaghettification") on the way in.
5. Entering energetic electron-neutrinos can interact with neutrons to produce protons and electrons ($\nu_e + n \rightarrow p + e$). Assume that all neutrinos either do this or become part of the rotating ring torus. The electron neutrino mass ≦0.120 eV/$c^2$is negligible compared to the three quarks' and electron's mass.
6. Strong gravity inside the EH can overcome the binding energy of the quarks in nucleons
7. Nucleons are eventually deconstructed into their constituent u and d "colored" quarks as they spiral closer to the ring torus. The u and d quarks and electrons are fermions, so individually cannot occupy the same quantum state (Pauli Exclusion Principle, PEP). It is assumed that the PEP is not violated in strong gravity.
8. Photons that enter the EH transfer their energy to the quarks and electrons contributing to the rotational motion of the quarks and electrons in the ring torus or join in that motion.
9. The unstable fundamental particles (See appendix below.) can be created by high-energy collisions of matter in the accretion disk circulating around the BH; these particles can enter the EH, but they should decay quickly into the stable fundamental particles.
10. Inside a BH the quarks and electrons, and perhaps electron neutrinos and photons, will fill a ring torus at the center of the black hole, which ring torus plane is orthogonal to the spin direction with each fermion fundamental particle type in a unique quantum state. The photons can occupy the same Planck volume because they are bosons. The quarks of a specific color, electrons and electron neutrinos cannot occupy the same Planck volume because they are fermions. The three colored quarks and electrons ring tori may be of different sizes. The very small ring torus replaces the ring singularity of the Kerr Model at the center of the BH.
11. The event-horizon (EH) equatorial radius equation for the Kerr Model is $\boxed{r_{EH} = \frac{1}{2}\left(r_S + \sqrt{r_S{}^2 - 4a^2}\right)}$, where $\boxed{a \equiv \frac{J}{cM}}$ is a spin parameter and $r_S \equiv \frac{2GM}{c^2}$ is the Schwarzschild EH radius. (To prevent a "naked singularity": $0 \le \sigma \le G/c^2$ where $\sigma \equiv a/M$.) Note that $r_{EH} = r_s$ for zero spin ($J = 0 \Rightarrow \sigma = 0$), the Schwarzschild-model case, and r_{EH} is a complex number for $r_S = \frac{GM}{c^2} < a = \sigma M \Rightarrow \sigma > \frac{G}{c^2}$; i.e., $\sigma = G/c^2$ defines the maximum possible spin. In 3D the EH is an oblate ellipsoid

$$\boxed{x^2 + y^2 + \left(\frac{2GM}{c^2 r_{EH}}\right) z^2 = \frac{2GM}{c^2} r_{EH}} \text{ where } r^2 + a^2 = 2r\frac{GM}{c^2} \text{ or } r_{EH} = \frac{GM}{c^2} + \sqrt{\left\{\frac{GM}{c^2}\right\}^2 - a^2}.$$

12. Consider a stellar BH with mass = $50M_\odot$ ($M_\odot$ = solar mass = 1.989 × 10^{30} kg), the average mass of the LIGO-O2 remnant masses, which would have an approximate Schwarzschild-event-horizon radius = 300 km. The surface gravity is proportional to 1/ $M_\odot$. For such a small BH entering atoms begin being deconstructed to their basic constituents before crossing the event horizon. The average Ligo-O2 spin parameter is σ=0.7; its maximum value is 1.
13. As shown in the appendices "Standard Model of Elementary Particles" and "Stable Isotopes", out of 108 quarks in average atoms (neutron/protons ratio = 7/5) 17 are "red" u-quarks, 17 are "green" u-quarks, 17 are "blue" u-quarks, 19 are "red" d-quarks, 19 are "green" d-quarks, 19 are "blue" d-quarks, accompanied by 15 electrons to balance the electric charge, with an average fundamental-particle mass of M_{avg} = (3 x 17 x 2.2 + 3 x 19 x 4.7 + 15 x 0.511)MeV**/c²/(** 3 x 17 + 3 x 19 + 15)= 5.6 x 10^{-26} kg. The quarks come in three "color" quantum numbers in protons and neutrons in atoms, so there are 7 overlapping ring tori for the 3 colored u-quarks with 2/3 electric charge, the 3 colored d-quarks with -1/3 electric charge and the electrons with -1 electric charge, and perhaps one more ring torus for electron-neutrinos. The electrons and d-quarks are distributed around the u-quarks to cancel out electric charge. As mentioned elsewhere energetic electron neutrinos can convert neutrons to protons and electrons, so the 17, 19 and 15 numbers mentioned above will not accurately represent the ratios of the non-zero-mass fundamental particles. However, the number given here are used as an approximation.
14. Assume that the kinetic energy of the incoming quarks, electrons, electron neutrinos and photons go into creating the rotation of the ring torus to achieve the spin J; see below. The mass M_{BH} of the black hole includes the masses of the fundamental particles plus the ring-torus' kinetic-energies/c^2.
15. In the Standard Model of Elementary Particles it is assumed that the fundamental particles are point particles. Instead, assume here that they each occupy a **"Planck" cube** with Planck-Length sides $l_P = 1.616229\times10^{-35}\,m$. The Planck cube volume = 4.221907 x 10^{-105}m^3. This assumes that space is continuous, so Planck spheres cannot be used; the simplest spherical geometry to fill space with space-quanta is a Planck cube.
16. Each of the 3 different-color d-quarks cannot occupy the same space because they are fermions. The 3 different color d-quarks' spaces can overlap. Similarly, each of the 3 different-color u-quarks cannot occupy the same space, which total space is about 17/19 smaller than the d-quarks' space. The 3 different color u-quarks' spaces can overlap. The u-quarks' spaces can overlap the larger d-quarks' spaces. The electrons cannot be in the same space because they are fermions, but their space will be about 15/17 smaller than the u-quarks' space. So, the maximum space volume in the ring torus will be the volume of the any-color d-quarks space.
17. So, for a $10M_\odot$ -BH ($M_\odot = 1.989\times10^{30}$ kg), the number of fundamental particles in the BH is about

$$N_{FunPart} = \frac{50M_\odot}{M_{avg}} = \frac{50\left(1.989\times10^{30}\ \text{kg}\right)}{5.6\times10^{-26}\ \text{kg}} = 17.75\times10^{56}$$ (There are about 10^{86} protons in the observable universe.) The total cube volume of the ring torus space, which is the volume for the any-color d-quarks, is:

$$V = \frac{10M_\odot}{M_{avg}}V_{Planck} = \frac{50\left(1.989\times10^{30}\ \text{kg}\right)}{5.6\times10^{-26}\ \text{kg}}\left(4.221907\times10^{-105}\text{m}^3\right) = 7.5\times10^{-48}\text{m}^3$$, practically a "ring singularity".

The largest any-color d-quark cube defines the size of the ring torus at the center of the 10 $M_\odot$ -BH. The volume of the largest torus is

$$V = \left(2\pi r_c\right)\left(\pi r_t^2\right) = 2\pi^2 r_t^2 \text{r}_c \text{ where}$$

r_t = radius of the torus tube and r_c = distance from the center of BH to the center of the torus tube.

For the largest volume, any-color d-quark:

$$V = 1.5 \times 10^{-48} m^3 = 2\pi^2 r_t^2 r_c.$$

This is a very thin ring torus.

Features

A question: Is the Pauli Exclusion Principle (PEP) stronger than the huge gravitation force that crushed matter into a black hole?

Conclusion

The model for the interior of a spinning black hole described in this article has the following properties:

1. Matter than enters the event horizon of a spinning black hole is eventually deconstructed into the stable fundamental particles of the Standard Model of particle physics, which are assumed to be unable to be broken into more fundamental units, even by very strong gravity.
2. The ring singularity of the Kerr Model of a black hole is replaced by a very thin rotating ring torus, which is very thin but not a singularity.
3. The ring torus contains the stable fundamental particles of the Standard Model of particle physics: quarks, electrons and perhaps electron neutrinos and photons.
4. The fundamental particles are cubes with Planck-Length sides packed together in the ring torus.
5. The rotation of the ring torus provides the spin of the black hole. (The spins of the remnant black holes detected by LIGO are all close to $J = 0.7\frac{GM^2}{c}$.)

This model surely has mathematical problems that perhaps no one can currently solve. However, it makes more physical sense to the author than does a ring singularity of uncertain character at the center of a rotating black hole as given by the Kerr Model.

If indeed the event horizon of a black hole does not allow any matter or energy out, this or any other model of a black hole cannot be proven correct or incorrect. (The miniscule thermal radiation of a black hole tells nothing about what the interior structure is.)

Appendix: Units of Astronomy

	AU = Sun to Earth	Light Year	km	Parsec
AU = Sun to Earth	1	1.58125×10^{-5}	1.496×10^{8}	4.8481×10^{-6}
Light Year	6.32411×10^{4}	1	9.461×10^{12}	0.306601
km	6.68459×10^{-9}	1.057×10^{-13}	1	3.24078×10^{-14}
Parsec	2.06265×10^{5}	3.26156	3.086×10^{13}	1

Astronomical Unit = AU =

"A unit of length, equal to the mean distance of the earth fromthe sun: approximately 93 million miles (150 million km)"

Light Year = ly =

"The distance traversed by light in one mean solar year, about 5.88 trillion mi. (9.46 trillion km): used as a unit in measuring stellar distances."

Parsec = pc =

"A unit of distance equal to that required to cause aheliocentric parallax of one second of an arc, equivalent to 206,265 times the distance from the earth to the sun, or 3.26light-years."

Appendix: Planck and Solar Units

Planck Units are defined in terms of physical constants:

Gravitational Constant G	Speed of Light c	Reduced Planck Constant $\hbar$
6.67408×10^{-11} m³·kg⁻¹·s⁻²	2.99792458×10^{8} m·s⁻¹	$1.054571800\times10^{-34}$ J·s

[**Units**]:

$$\left[G\frac{M}{c^2}\right]=\frac{m^3}{kg\cdot s^2}\frac{kg\cdot s^2}{m^2}=m=[R]$$

$$\left[G\frac{M^2}{c}\right]=\frac{m^3}{kg\cdot s^2}\frac{kg^2\cdot s}{m}=\frac{kgm^2}{s}=[MVR]=[J]$$

The **Planck Length**: $\boxed{l_P\equiv\sqrt{\frac{\hbar G}{c^3}}=1.616229\times10^{-35}\,m}$ and the **Planck Mass**: $\boxed{m_P\equiv\sqrt{\frac{\hbar c}{G}}=2.176471\times10^{-8}\,kg}$.

Sun: Mass ≡ $M_\odot$ = 1.988500 x 10^{30} kg; Radius ≡ $R_\odot$ = 6.95700 x 10^5 km; $GM_\odot$ = 1.32712 x 10^{11} km³/s²;

Solar Schwarzschild EH radius = $r_S\equiv\frac{2GM_\odot}{c^2}=2.95324\text{ km}$

Solar Kerr EH radius: $r_{EH}=\frac{1}{2}\left(r_S+\sqrt{r_S^2-4a^2}\right)$, where $\boxed{a\equiv\frac{J}{cM}}$ and $\sigma\equiv\frac{a}{M}=\frac{J}{cM^2}$.

σ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
r_{EH}	2.95324	2.902364	2.849603	2.794733	2.737477	2.677494	2.614353	2.547495	2.476176	2.39936	2.31554

$$\sigma\equiv\text{in units}\left[\frac{cJ}{GM^2}\right];\ \sigma_{\max}=1;\ J_{\max}\text{in units}\left[\frac{GM^2}{c}\right].$$

To get EH for any BH multiply the values for a solar BH by the new BH mass in units of the solar mass $M_\odot$.

E.g., the SGR A*, the supermassive BH at the center of the Milky Way galaxy has a mass of (4.02± 0.16) x 10^6 $M_\odot$.

So, its $r_S=\left(4.02\times10^6\right)\frac{2GM_\odot}{c^2}=\left(4.02\times10^6\right)\left(2.95324\text{ km}\right)=11.9\times10^6\text{ km}=1.25\times10^{-6}$ light-years.

If it has spin parameter σ = 0.7 its $r_{EH}=\left(4.02\times10^6\right)\left(2.547495\right)=10.2\times10^6\text{ km}=1.08\times10^{-6}$ light-years.

Appendix: Torus

The equation of a ring torus azimuthally symmetric about the z axis:

$$\boxed{\left(a-\sqrt{x^2+y^2}\right)+z^2=c^2},$$

Where $a=$radius from the center of the hole to the center of the torus tube and $c=$ radius of the tube.

For a ring torus $c<a$.

The volume of a ring torus is:

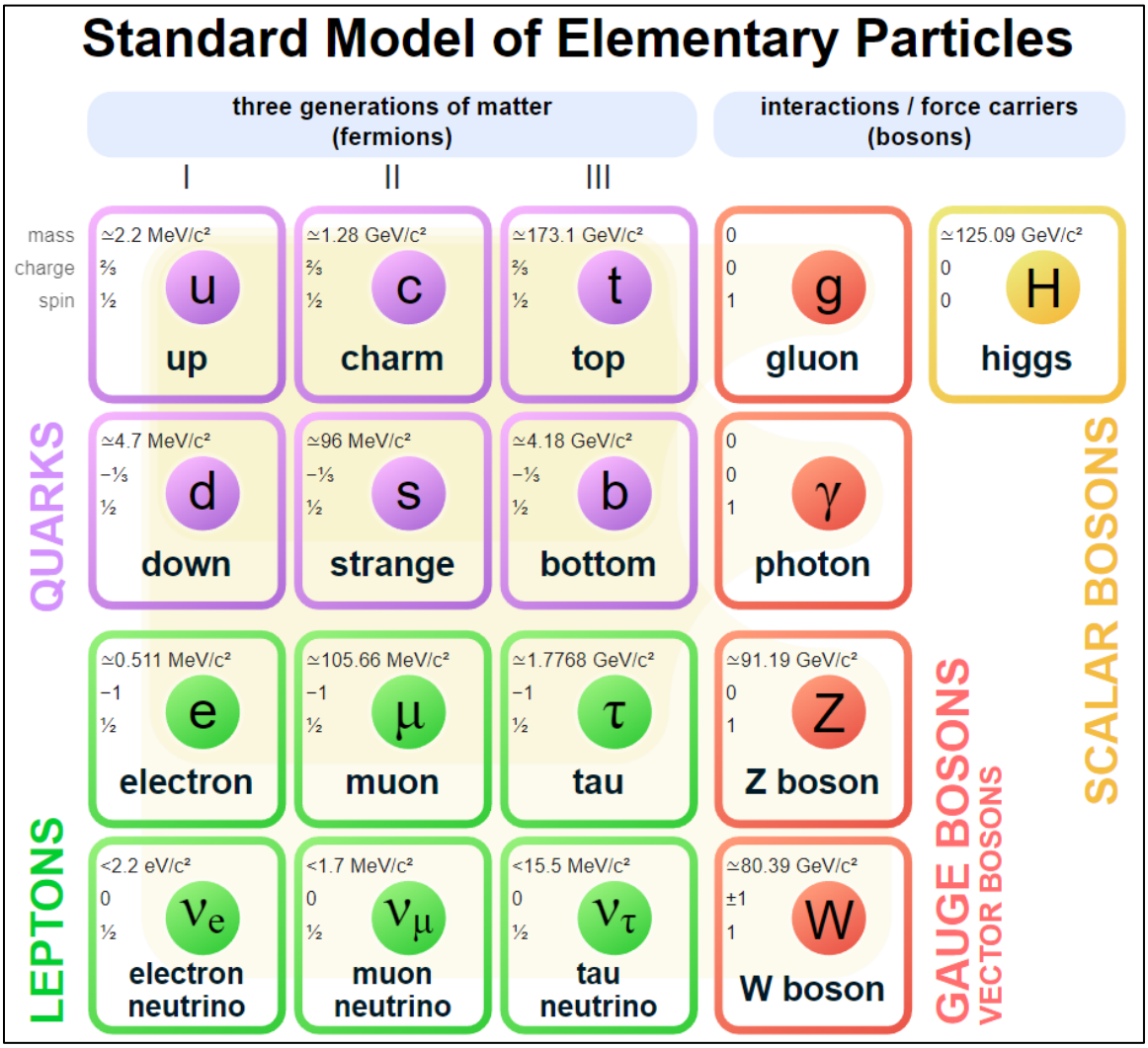
$$V=\text{(area of torus cross section) x (circumference of tube)}=\left(\pi c^2\right)\left(2\pi a\right)=2\pi^2c^2a\ .$$

The surface area of a ring torus is:

$$S=\text{(circumference of cross section) x (circumference of tube)}=\left(2\pi c\right)\left(2\pi a\right)=4\pi^2ac\ .$$

Appendix: Standard Model of Particle Physics

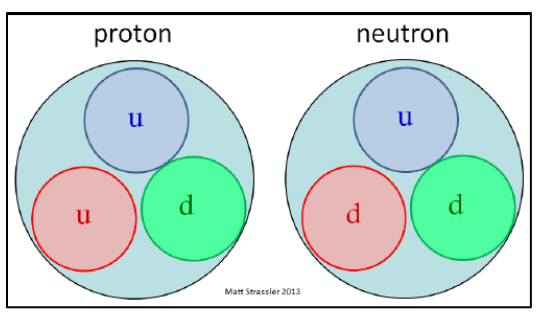
The fundamental particles of the Standard Model are:



The first column lists the stable fundamental fermions; the gluon and photon are the fundamental stable bosons.

1 MeV/c^2 = 1.782662695946 x 10^{-30} kg

Molecules on the Earth are made of atoms that have a nucleus made of protons and neutrons surrounded by electrons. The protons and nucleons (nucleons) are made of quarks with 3 different "colors":



Stable isotopes have an average ratio of neutrons to protons of about 7/5. (See appendix about "Stable Isotopes" below.) So, for 36 quarks about 17 are u-quarks (mass = 2.2 MeV/c^2 and electric-charge +2/3) and 19 are d-quarks (mass = 4.7 MeV/c^2 and electric-charge -1/3) and there are 5 electrons (mass = 0.511 MeV/c^2 and electric-charge -1) to cancel out the 5 quark positive electric charges. The mass of those 36 quarks and 5 electrons is:

(17 x 2.2 +19 x 4.7 + 5 x 0.511) MeV/c^2 = 129.3 MeV/c^2 = 2.30 x 10^{-28} kg.

(There are unproven theories that the quarks are not fundamental because there are "preons" as constituents inside quarks.)

Appendix: Relativistic Kinematics of the Ring Torus

- Assume that the ring torus tube's cross section is circular with radius r_t and the center of the circle is at radius r_c from the center of the black hole (BH).
- Assume that the black-hole mass (M_{BH}) is known and all of it is in the ring torus.
- Take $M_{BH} = 50M_{\odot}$, the average LIGO-O2 remnant mass of binary black-home mergers, as an example.
- Assume that relativistic kinematics applies to the ring singularity.
- The BH mass: $M_{BH}c^2 = K + M_{BH0}c^2 \equiv K + \sum_{quarks} M_q c^2 + \sum_{electrons} M_e c^2 + \sum_{eneutrinos} M_e c^2$ where

  M_{BH0} = **BH rest mass** of the matter in the ring torus. The kinetic energy of the masses =

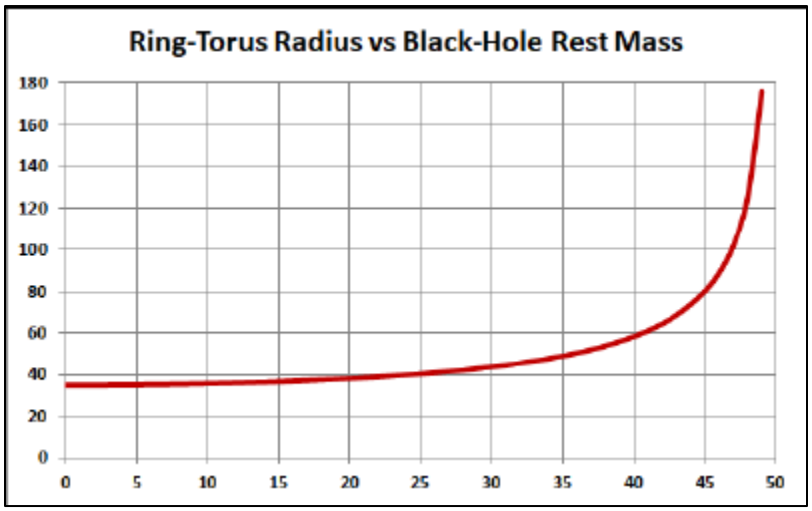
  $K \equiv K_{matter} + K_{photons} = M_{BH}c^2 - M_{BH0}c^2 = (\gamma - 1)M_{BH0}c^2$, where $\gamma \equiv 1/\sqrt{1 - v^2/c^2}$.
- Therefore, $\boxed{v = c\frac{\sqrt{M_{BN}^2 - M_{BH0}^2}}{M_{BH}}}$. Limits: If $M_{BH0} = M_{BH} \Rightarrow v = 0$; $\boxed{\text{if } M_{BH0} = 0 \Rightarrow v = c}$.

  $M_{BH0} = 0$ means that the ring singularity is entirely made of photons and/or other zero-mass particles.
- Thus, the BH ring-torus spin is: $\boxed{J = M_{BH} v r_c = c\sqrt{M_{BN}^2 - M_{BH0}^2}\, r_c}$. Then $\boxed{c\sqrt{M_{BN}^2 - M_{BH0}^2}\, r_c = \sigma \frac{GM_{BH}^2}{c}}$,

  where $\boxed{\sigma \equiv \frac{cJ}{GM_{BH}^2}}$ defines a dimensionless spin parameter σ for which $0 \le \sigma \le 1$; the upper limit is to prevent a naked singularity.
- There are two unknowns, r_c and M_{BH0}, the radius of the ring torus from the center of the BH and the rest mass of the quarks + electrons + electron neutrinos in the ring torus. Solve for $\boxed{r_c = \frac{\sigma G M_{BH}^2}{c^2\sqrt{M_{BN}^2 - M_{BH0}^2}}}$.
- Solve for $\boxed{M_{BH0} = \frac{M_{BH}}{c^2 r_c}\sqrt{c^4 r_c^2 - \sigma^2 G^2 M_{BH}^2}}$. Note that $M_{BH0} = 0$ when $r_c = \sigma \frac{GM_{BH}}{c^2}$.

  Use as an example: $M_{BH} = 50M_{\odot}$ and $\sigma = 0.7$ for the average remnant-BHs of LIGO-O2 binary BH mergers given in an appendix: $r_c = \sigma \frac{GM_{BH}}{c^2} = 35\frac{GM_{\odot}}{c^2}$ for $M_{BH0} = 0$.

- Plot r_c vs M_{BH0} with $\sigma = 0.7$, M_{BH} =50$M_\odot$ and units $[r_c] = GM_\odot / c^2$:



Limits: $r_c = \sigma \dfrac{GM_{BH}}{c^2} = 35\dfrac{GM_\odot}{c^2}$ for $M_{BH0} = 0$ and $r_c = \infty$ for $M_{BH0} = M_{BH}$.

Compare to the event horizon: $\boxed{r_{EH} = \dfrac{1}{2}\left(r_S + \sqrt{r_S^{\,2} - 4a^2}\right)}$ where $r_S \equiv \dfrac{2GM_{BH}}{c^2}$ and $\boxed{a \equiv \dfrac{J}{cM_{BH}}}$ and $\boxed{\sigma \equiv \dfrac{cJ}{GM_{BH}^2}}$:

$$\therefore r_{EH} = \frac{1}{2}\left(\frac{2GM_{BH}}{c^2} + \sqrt{\left(\frac{2GM_{BH}}{c^2}\right)^2 - 4\left(\frac{J}{cM_{BH}}\right)^2}\right) = \frac{GM_{BH}}{c^2} + \sqrt{\left(\frac{GM_{BH}}{c^2}\right)^2 - \left(\sigma\frac{GM_{BH}}{c^2}\right)^2}$$

$$= \frac{GM_{BH}}{c^2}\left(1 + \sqrt{1-\sigma^2}\right) = \boxed{85.7\frac{GM_\odot}{c^2} = r_{EH} \text{ for } M_{BH} = 50M_\odot \text{ and } \sigma\text{=0.7}} \text{ from LIGO-O2 averages.}$$

$$\text{If } r_c = r_{EH} = \frac{\sigma GM_{BH}^2}{c^2\sqrt{M_{BN}^2 - M_{BH0}^2}} = \frac{1750}{\sqrt{2500M_\odot - M^2{}_{BH0}}}\frac{GM_\odot}{c^2} = 85.7\frac{GM_\odot}{c^2} \Rightarrow M_{BH0} = 45.6M_\odot$$

The event horizon radius must be greater than the radius of the ring torus:

$$\text{For } r_c < r_{EH} = 85.7\frac{GM_\odot}{c^2} \Rightarrow M_{BH0} < 45.6M_\odot \text{ for a BH with } M_{BH} = 50M_\odot \text{ and spin parameter } \sigma = 0.7.$$

For a BH of mass M_{BH} =50$M_\odot$ and spin parameter $\sigma = 0.7$:

$$\boxed{35\frac{GM_\odot}{c^2} \le r_c < 85.7\frac{GM_\odot}{c^2}} \text{ and } \boxed{0 \le M_{BH0} < 45.6M_\odot}.$$

Since about 99% of the mass of nucleons is binding energy of the constituent quarks, r_c and M_{BH0} must be near or at their lower limits.

The Kerr Model has a ring singularity at $r_c = a$. The average spin and BH mass for the LIGO-O2 binary BH mergers are $\sigma = 0.7$ and M_{BH} =50M$_\odot$ where $\sigma = \frac{cJ}{GM_{BH}^2}$ and $a = \sigma \frac{GM_{BH}}{c^2}$. So, $\boxed{r_c = \sigma \frac{GM_{BH}}{c^2} = 0.7 \frac{GM_{BH}}{c^2} = 35 \frac{GM_\odot}{c^2}}$ the lower limit for $M_{BH0} = 0$ given above. This kinematic calculation applies whether the Kerr Model or the Ring-Torus Model is considered. So**, for the Kerr Model the mass of the ring-singularity's mass is totally due to kinetic energy and the circular speed is** $v = c$ **!** So, this could be just photons orbiting at speed c. In that case, apparently the "fundamental particles" quarks and electrons are crushed out of existence or never entered the event horizon! Since the innermost orbit outside the EH is the co-rotating photon orbit (See Appendix.), it makes sense that only photons enter the EH.

Appendix: LIGO-O2 Results

Event Name	Merger	Distance (Giga light years)	Energy Radiated ($c^2M_\odot$)	Primary Mass ($c^2M_\odot$)	Secondary Mass ($c^2M_\odot$)	Remnant Mass ($c^2M_\odot$)	Spin (σ=cJ/GM^2)	Energy Emitted
GW170608	BH+BH->BH	1.109	0.85	12	7	18	0.69	GW
GW170608	BH+BH->BH	1.044	0.9	10.9	7.6	17.8	0.69	GW
GW151226	BH+BH->BH	1.435	1.0	13.7	7.7	20.5	0.74	GW
GW170104	BH+BH->BH	3.132	2.2	31.0	20.1	49.1	0.66	GW
GW170814	BH+BH->BH	1.892	2.7	30.7	25.3	53.4	0.72	GW
GW170809	BH+BH->BH	3.229	2.7	35.2	23.8	56.4	0.70	GW
GW170818	BH+BH->BH	3.327	2.7	35.5	26.8	59.8	9.67	GW
GW150914	BH+BH->BH	1.403	3.1	35.6	30.6	63.1	0.69	GW
GW170823	BH+BH->BH	6.034	3.3	39.6	29.4	65.6	0.71	GW
GW170729	BH+BH->BH	8.971	4.8	50.6	34.3	80.3	0.81	GW
GW170817	NS+NS->NS	0.130	0.04	1.46	1.27	≦2.8	≦0.89	GW+Kilonova
Future:	BH+NS->BH							GW+Kilonova

BH = Black Hole; NS = Neutron Star

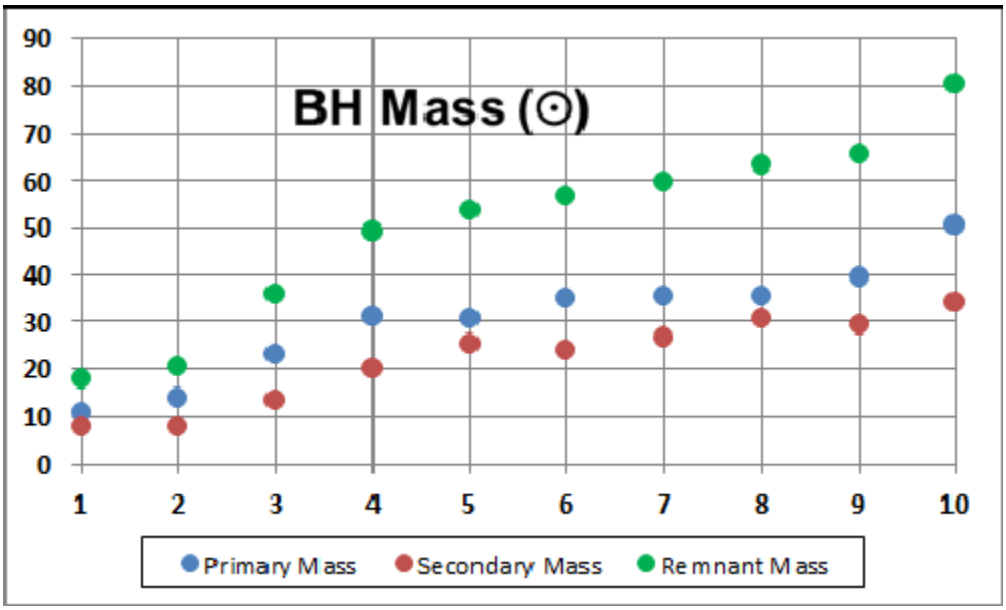
1 light year = 9.46 x 10^{15} meters; Age of universe = 13.8 gigayears.
In 5 gigayears the Sun has radiated 0.0003 (c^2M_S) energy.

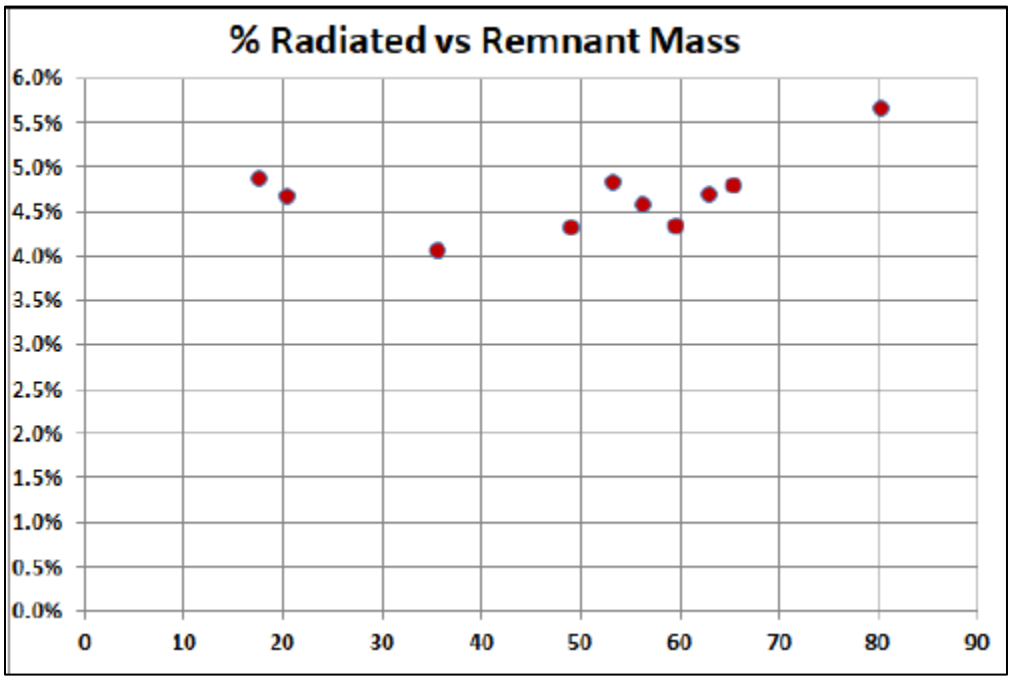
Average spin parameter = 0.706, median spin parameter = 0.695; standard deviation = 0.049

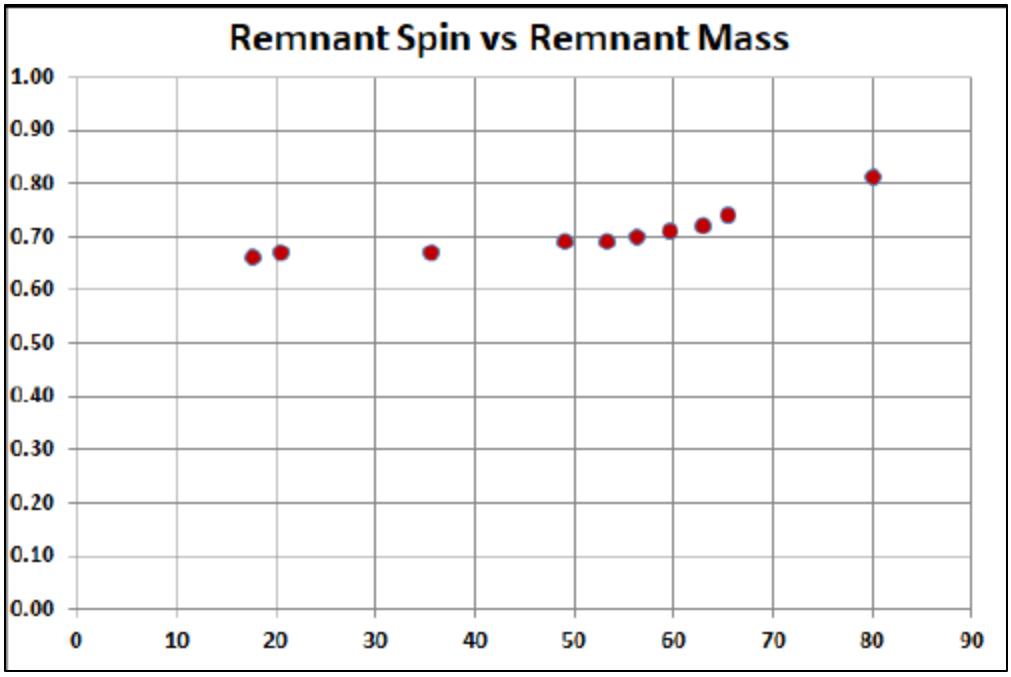
LIGO-O2 Remnant Spins $\sigma = \frac{cJ}{GM_{BH}^2}$ and mass in solar masses $M_\odot$:

Binary BH	1	2	3	4	5	6	7	8	9	10	Average	Median	Standard Deviation
Remnant Mass ($M_\odot$)	18	17.8	20.5	49.1	53.4	56.4	59.8	63.1	65.6	89.3	50.2	54.9	20.0
Spin (cJ/GM^2)	0.69	0.67	0.74	0.66	0.69	0.81	0.7	0.72	0.67	0.71	0.706	0.695	0.044

Binary and Remnant Black-Hole Masses:







Average spin parameter = 0.706, median spin parameter = 0.695; standard deviation = 0.049

Appendix: Photon Orbits for Kerr Model of Black Holes

The co-rotating-photon orbit is in the equatorial plane of radius:

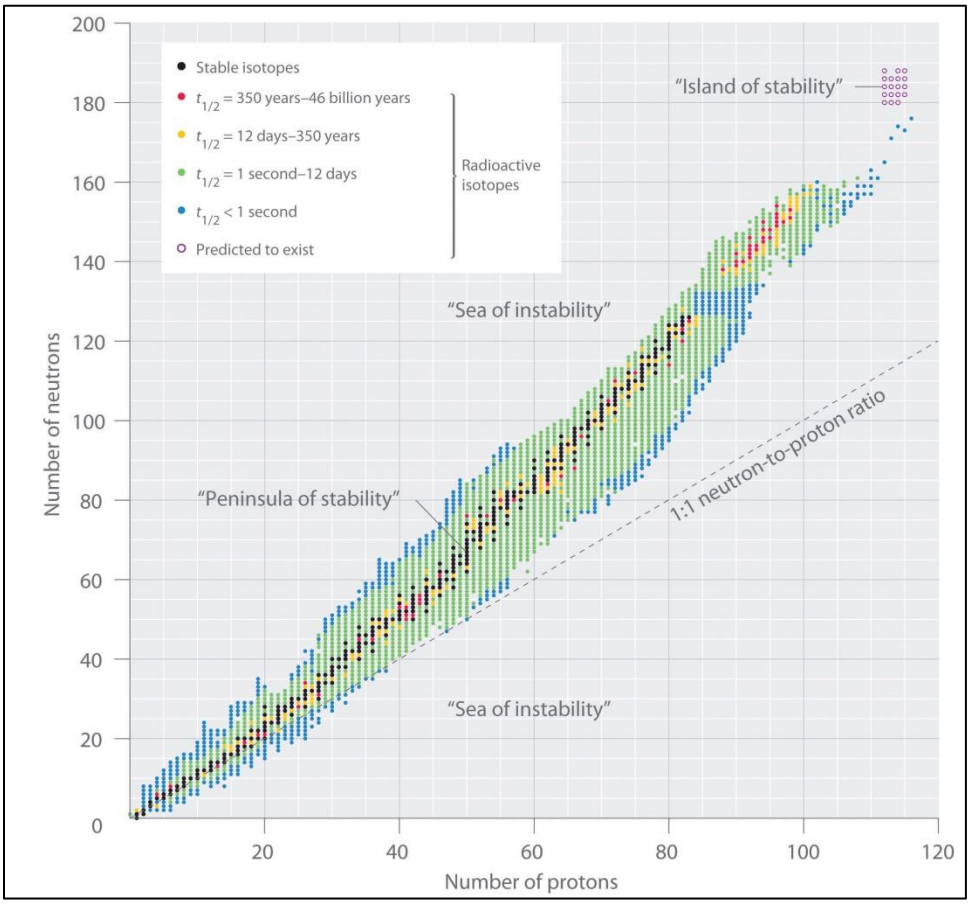
$$r_{-} = \frac{2GM}{c^2}\left[1+\cos\left\{\frac{2}{3}\cos^{-1}\left(-|\sigma|\right)\right\}\right] \text{ where } \sigma \equiv \frac{cJ}{GM_{BH}^2}.$$

The range is $\frac{GM_{BH}}{c^2} \le r_p \le \frac{3GM_{BH}}{c^2}$ for $1 \ge \sigma \ge 0$.

There is a contra-rotating photon orbit farther out: $r_{+} = \frac{2GM}{c^2}\left[1+\cos\left\{\frac{2}{3}\cos^{-1}\left(|\sigma|\right)\right\}\right]$

The range is $\frac{3GM_{BH}}{c^2} \le r_p \le \frac{4GM_{BH}}{c^2}$ for $0 \le \sigma \ge 1$.

Appendix: Stable Isotopes



The average ratio of neutrons to protons in stable isotopes is about 7/5