LIGO Binary Black Hole Mergers Waveform Phenomenology

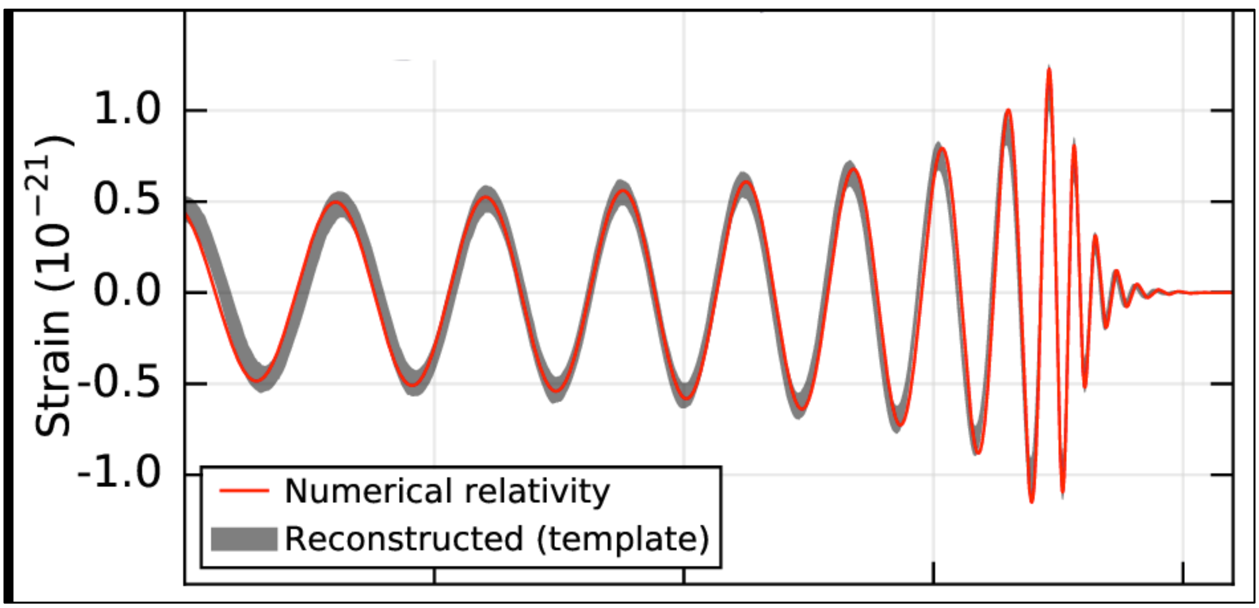
L. David Roper, ROPERLD@VT.EDU 30 April 2019

Introduction

This is a study of the numerical-relativity calculation of the waveform of the LIGO-O2 binary-black-hole merger GW150914. Some interesting parameters of the merger are:

Event Name	Distance (Gly)	Energy Radiated (c^2MS)	Primary Mass	Secondary Mass	Remnant Mass	Spin (cJ/GM^2)
GW150914	**1.403**	**3.1**	**35.6**	**30.6**	**63.1**	**0.72**

When I first saw the LIGO Binary Black Holes Mergers' waveforms I immediately thought that they look like a single sine wave with variable amplitude and variable frequency. So, I decided to see if I could fit a modulated mathematics sine function to one of the waveforms; I chose the GW150914 waveform. Here is its numerical-relativity calculated "estimated gravitational-wave strain amplitude":



The left horizontal tic edge is 0.25 sec and the last horizontal tic mark is 0.45 sec.

The calculation of binary-black-holes-merger waveforms involves three time regions:

1. Inspiral post-newtonian calculation
2. Numerical relativity merger calculation
3. Remnant black hole ringdown calculation

The three calculated waveforms are matched at the two time boundaries:

1. When the innermost stable circular orbits (ISCO) of the two black holes touch for calculations 1 and 2.
2. When the two event horizons touch and become the remnant black hole event horizon for calculations 2 and 3.

From Physics Today:

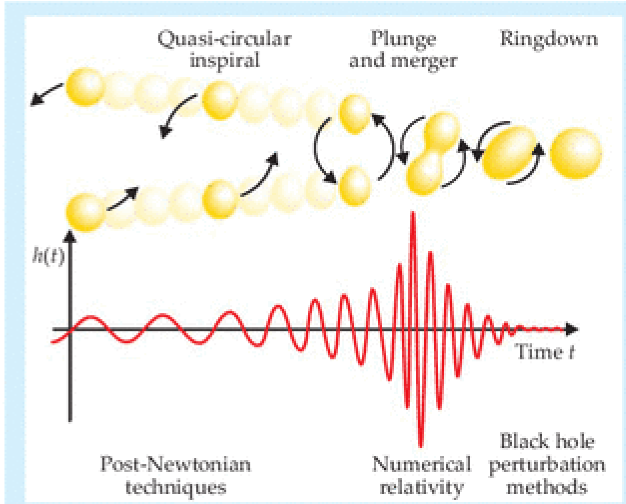
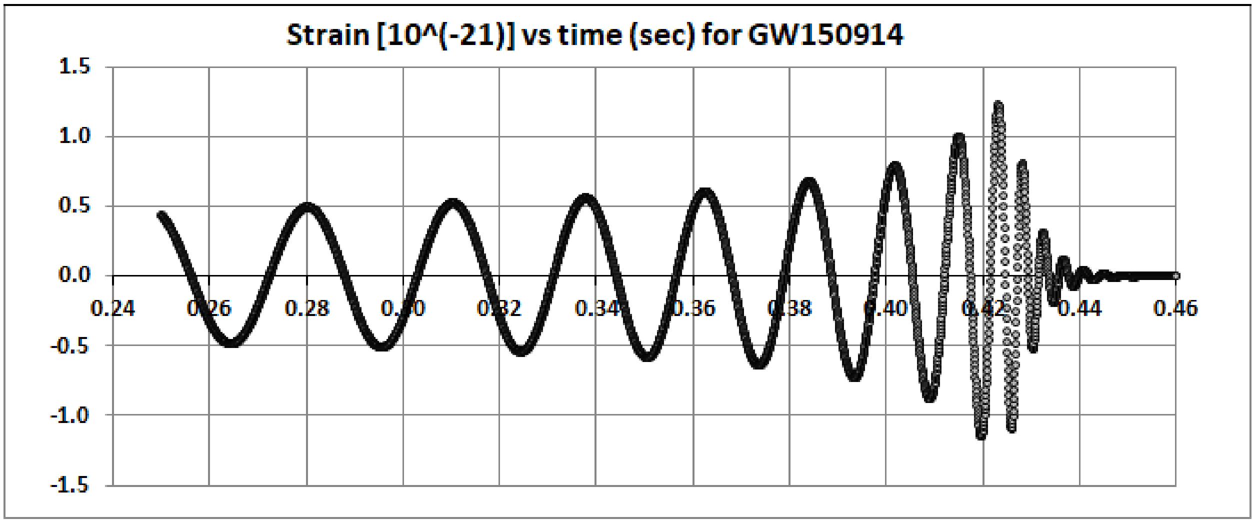


Figure 1. Coalescence of a compact binary. The loss of energy and angular momentum via the emission of gravitational radiation drives compact-binary coalescence, which proceeds in three different phases. The strongest gravitational-wave signal, illustrated here as the gravitational-wave amplitude h, accompanies the late inspiral phase and the plunge and merger phase; for that part of the coalescence, post-Newtonian and perturbation methods break down, and numerical simulations must be employed. (Adapted from ref. 3.)

The purpose of this article is to fit the simplest possible mathematical equation to the waveform data shown above. If similar waveforms for the other nine binary black hole mergers of LIGO-O2 could be fitted with the same equation, perhaps correlations of the parameters of the fits could be made to some of the remnant-black-holes' parameters given below in the Conclusion.

Waveform Data Fitting

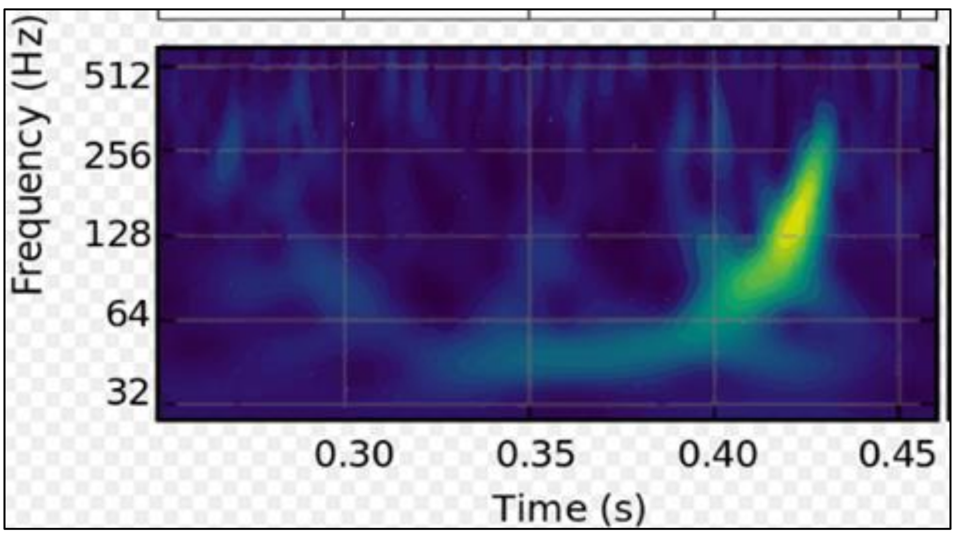
The waveform data points are available; here is a plot of them:



I tried various asymmetric peak functions for the varying peaked amplitude and settled on this simple one:

$$\boxed{Ex(t) \equiv \frac{C \exp\left(\frac{t-t_0}{\tau}\right)}{1+\exp\left(\frac{t-t_0}{n\tau}\right)} \text{ where } 0<n<1}.$$

It was difficult to get a digitized version of the frequency curve because the graph abscissa is logarithmic:



Noticing that the frequency was flat at first and then rose rapidly, I chose the two hyperbolic tangents because an exponential does not work:

$$\boxed{\text{Tanh}(t) \equiv \frac{1}{2}\left[a+c+(b-a)\tanh\left(\frac{t-t_b}{w_b}\right)+(c-b)\tanh\left(\frac{t-t_c}{w_c}\right)\right]}$$

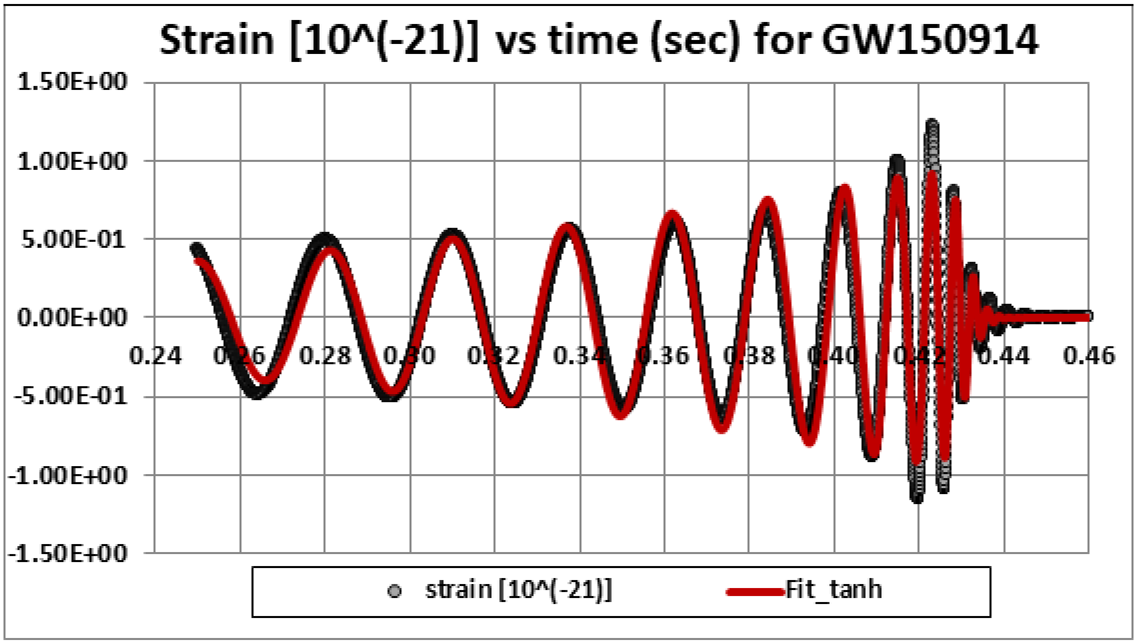
where $a=0$ is the starting flat level, b is the intermediate flat level and c is the final flat level and (w_b, w_c) are twice the rising exponential time constants.

The total function used to fit the data is:

$$\boxed{W(t) = Ex(t)\sin\left[2\pi \operatorname{Tanh}(t)(t-s)\right]}$$

The " s " parameter is the offset for the sine function.

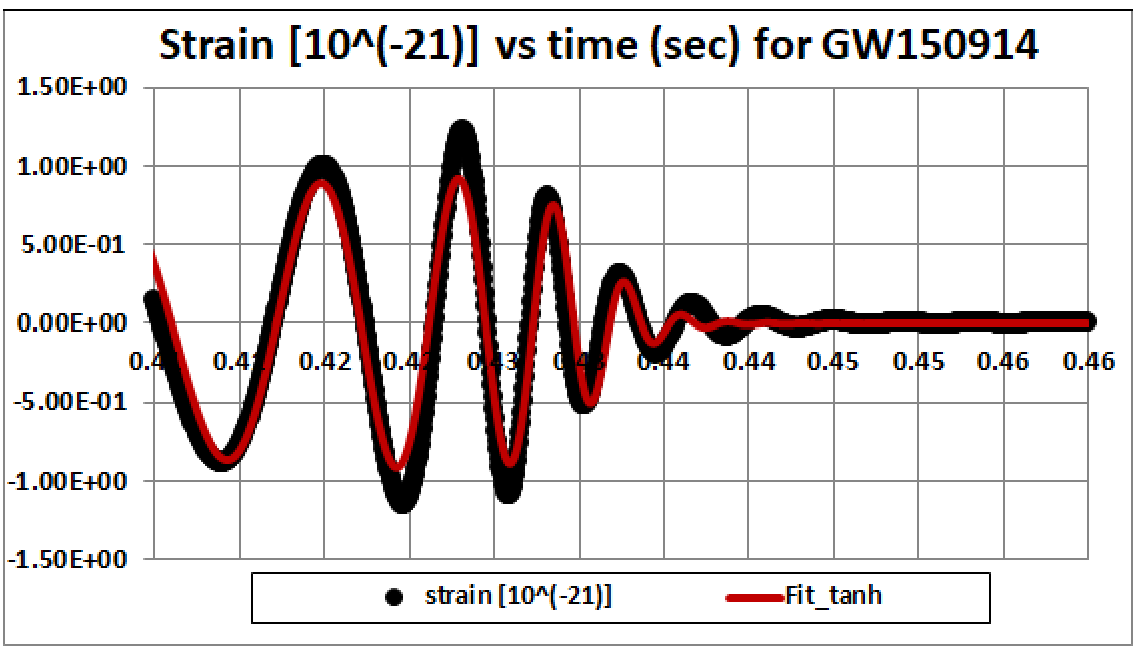
A least-squares fit of function $W(t)$ to the strain data yielded:



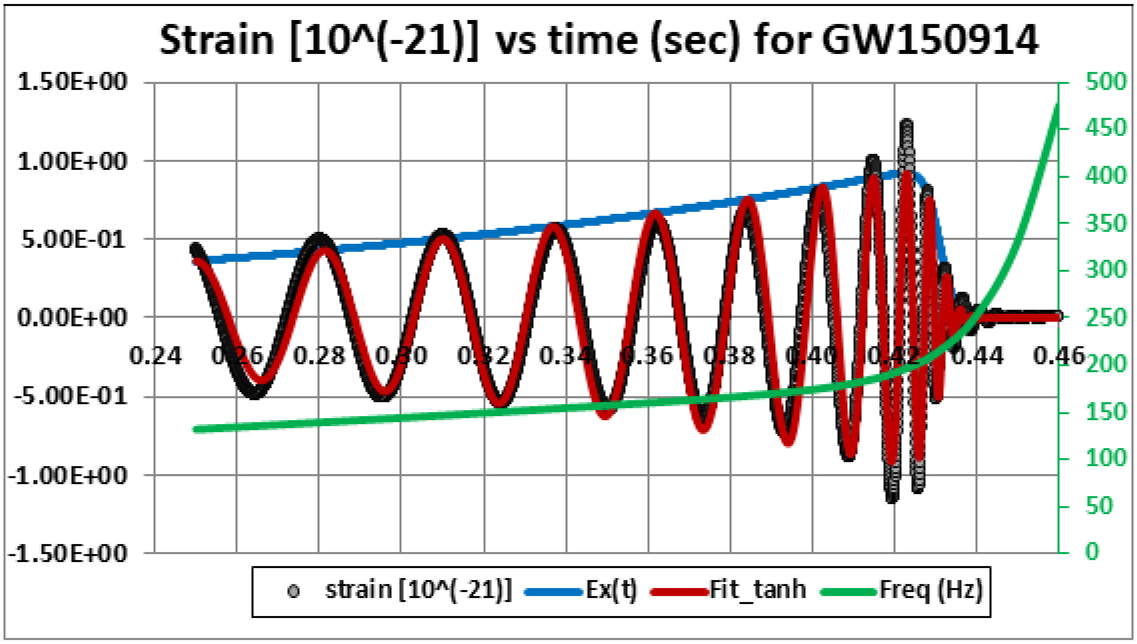
I was pleasantly surprised at how well the function fitted the data ($\chi^2 = 37.8$ for 3440 data).

The closeness of the fit may be good enough to derive correlations of the fit parameters with some of the LIGO-O2 parameters of the remnant-black-holes' mergers given below in the Conclusion.

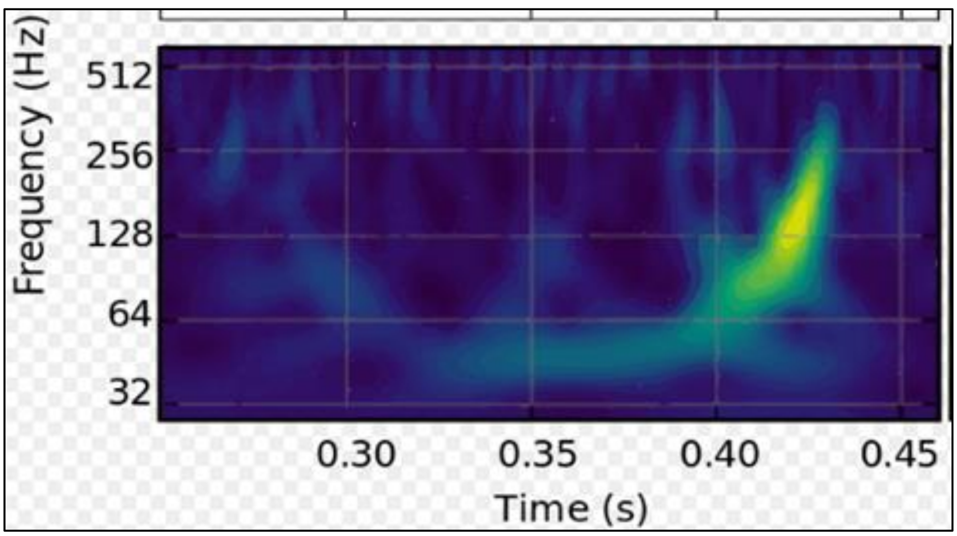
Here is a zoom in on the high-frequency end:



The following graph shows the varying amplitude function in blue and the varying frequency function in green.



I do not understand why the frequency function in the graph above does not match the **GW150914 frequency function:**



The initial flat frequency is much higher in the fit that in the graph above.

The fit parameters are:

	Ex	Chi.Sq	37.83221
C	0.97384		tanh
to	0.430848	a	0
τ	0.182553	b	82.46395
n	0.01	tb	0.691227
Sin	0	wb	0.822922
		c	332.9123
		tc	0.479261
		wc	0.025716

The "**s**" parameter is the offset for the sine function.

Conclusion

I would expect that numerical gravity would yield curves that are much more complicated that a simple amplitude and frequency modulated sine wave. I would think it would be a complicated Fourier series. The fact that it is so simple must mean that the complicated theory reduces to some simple phenomenology, as sometimes happens in physics.

I will be interesting to do this fit for other binary black holes mergers to see if some correlations can be made between the mergers' parameters and the fit parameters. Here are some parameters of the mergers:

Event Name	Distance (Gly)	Energy Radiated (c2MS)	Primary Mass	Secondary Mass	Remnant Mass	Spin (cJ/GM2)
GW170608	1.044	0.9	10.9	7.6	**17.8**	**0.69**
GW151226	1.435	1.0	13.7	7.7	**20.5**	**0.67**
GW151012	3.458	1.5	23.3	13.6	**35.7**	**0.74**
GW170104	3.132	2.2	31.0	20.1	**49.1**	**0.66**
GW170814	1.89196	2.7	30.7	25.3	**53.4**	**0.69**
GW170809	3.22938	2.7	35.2	23.8	**56.4**	**0.81**
GW170818	3.32724	2.7	35.5	26.8	**59.8**	**0.7**
GW150914	1.403	3.1	35.6	30.6	**63.1**	**0.72**
GW170823	6.0347	3.3	39.6	29.4	**65.6**	**0.67**
GW170729	8.9705	4.8	50.6	34.3	**80.3**	**0.71**
Average	**2.365**	**2.100**	**26.988**	**19.438**	**44.475**	**0.710**
Median	**2.512**	**2.450**	**30.850**	**21.950**	**51.250**	**0.695**
Standard Dev.	**1.015**	**0.854**	**9.949**	**8.821**	**17.681**	**0.048**