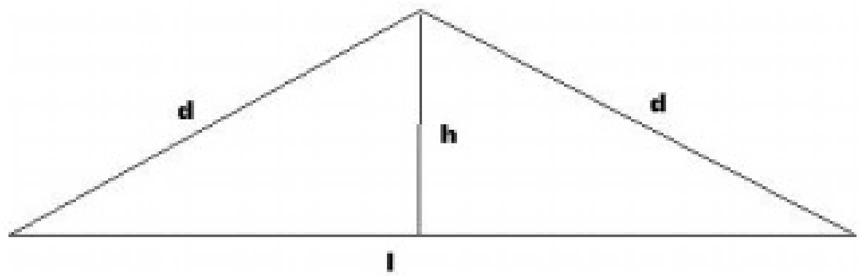
Energy Differences in Hilly Driving for Electric Vehicles:

Introduction

Consider the idealized hill (an isosceles triangle) shown in the box, with the left and right angle being θ .



Let $d =$ sides of the triangle and $l =$ the base of the triangle. Then $\boxed{h = \frac{l}{2}\tan\theta = d\sin\theta}=$ the perpendicular distance of the peak to the base.

Consider the case of **an electric vehicle traveling up and down the hill a distance $\mathbf{2d}$ at a constant speed** v and compare it to the case of **an electric vehicle traveling on the flat a distance $\mathbf{2d}$**. At constant speed, v, the kinetic energy, $\frac{1}{2}mv^2$, is constant, also.

The electric vehicle's energy, which comes from a battery, moves the vehicle up the hill. It will be shown that no battery energy is used to provide energy to move the vehicle **down** the hill at constant speed if the hill angle is greater than some value. Some fraction f of the available energy when the vehicle goes down the hill can be converted by a generator into electrical energy (regenerated) to charge the battery.

The **gravitational potential energy gained** going from the bottom to the top of the hill is $\boxed{E_g = mgh = mgd\sin\theta}$ and the same amount of gravitational potential energy is available going from the top to the bottom of the hill to convert into heat energy or charge the battery.

Assume that the only external frictional forces of consequence are the **air drag due to the velocity v of the vehicle and the rolling resistance of the tires and internal friction**.

For air drag:

$$\boxed{F_d = -\frac{1}{2}c_dA\rho v^2 \equiv -mCv^2}$$

where

- $c_d =$ drag coefficient (unitless),
- $A =$ projected area perpendicular to the velocity
- $\rho =$ air density $\approx 1.2\frac{\text{kg}}{\text{meter}^3}$ (Depends on the temperature, air pressure and the amount of water vapor in the air.),
- $m =$ mass of the vehicle (The mass is only needed in the equation directly below; it will cancel out in the final equations of interest.).
- $C \equiv \frac{1}{2}c_dA\rho/m$.

Example: Chevrolet Bolt EV:

- Area $=$ 2.34 m^2, mass $=$ 1625+80 (driver) kg,

$c_d = 0.312 \rightarrow C = \frac{1}{2}\frac{0.312\left(2.34\,\mathrm{m}^2\right)\left(1.2\frac{\mathrm{kg}}{\mathrm{m}^3}\right)}{1705\,\mathrm{kg}} = 2.6957 \times 10^{-4}\,\mathrm{m}^{-1}$

For rolling resistance of the tires):

$$\boxed{F_r = Rmg}.$$

The assumption is that the rolling resistance is proportional to the weight of the vehicle. Example: $F_r = 0.0075mg$. Then, for Bolt:

$F_r = 0.0075gm = 0.0075 \cdot 1705\,\mathrm{kg} \cdot 9.8\frac{\mathrm{m}}{\mathrm{s}^2} = 125.32\,\mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}^2} = 125.32\,\mathrm{J}$

Thus, the total dissipative force is $F = F_d + F_r$.

Traveling Over the Hill

Since the speed remains constant going up the hill, the vehicle's battery must provide the energy lost to heat by the **air drag and rolling resistance going up the hill** at constant speed v , which is

$$\boxed{E_{ud} = (F_d + F_r)d = m(Rg + Cv^2)d}\ .$$

Similarly, gravity provides the energy lost to heat by **air drag and rolling resistance going down the hill** at constant speed v , which is

$$\boxed{E_{dd} = (F_c + F_r)d = m(Rg + Cv^2)d}\ .$$

The **total energy that must be supplied by the vehicle's battery going up the hill** is the sum of the air-drag/rolling-resistance energy loss and the increase in gravitational potential energy:

$$\boxed{E_u = E_{ud} + E_g = m(Rg + Cv^2)d + mgd\sin\theta}\ ,$$

since the vehicle's kinetic energy remains constant going up the hill.

The **total energy change going down the hill** is

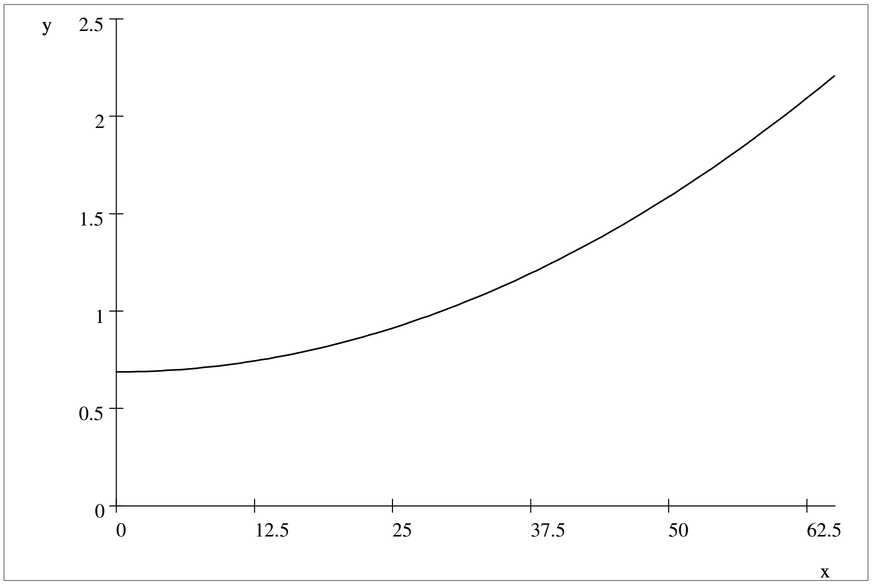
$$\boxed{E_d = E_g - E_{dd} = mgd\sin\theta - m(Rg + Cv^2)d}\ ,$$

since the vehicle's kinetic energy remains constant going down the hill.

That is, gravity provides energy and energy is lost due to air drag and rolling resistance. Note that if $mgd\sin\theta > m(Rg + Cv^2)d$, then the **vehicle energy change will be positive going down the hill**; simplified, this condition is

$$\boxed{g\sin\theta > Rg + Cv^2}$$

(Note that the equation yields v in $\frac{\text{meter}}{\text{s}}$; to get it in mph multiply by 2.237 . http://www.onlineconversion.com/speed_all.htm) If the hill angle $\theta < \arcsin[(Rg + Cv^2)/g]$, then the vehicle cannot travel down the hill at constant speed v; it will lose energy to heating the environment and, thus slow down as it goes down the hill. Then our condition of constant speed would not hold. If the hill angle $\theta < \arcsin(Cv^2/g)$, then the vehicle cannot travel down the hill at constant speed v; it will slow down as it goes down the hill. Here is a plot of the minimum angle (degrees) versus vehicle speed (mph) for the Nissan LEAF such that it can go down the hill at constant speed:



So, some energy is available to charge the system's battery going down the hill for hills of large enough angle depending on the speed. This extra energy beyond what is lost to air drag and rolling resistance could then be used to charge the battery to help move the vehicle over the next part of the trip.

Assume that some of the energy supplied by the battery going up the hill is recovered by charging the battery when going down the hill. Let $f =$ the fraction of gravitational energy (**regeneration**) provided by the battery going up the hill that is stored in the battery when going down the hill $(0 < f < 1)$. The energy stored in the battery going down the hill is $E_b = fE_u = fmgd\sin\theta$. Then the energy expended by the battery going over the hill is $E_h = E_u - E_b = dm[g\sin\theta + Rg + Cv^2] - fmgd\sin\theta$ or

$$\boxed{E_h = dm[Rg + Cv^2 + (1-f)g\sin\theta]}.$$

Note that some energy is lost due to heating the environment.

Traveling on the Flat

The energy expended by the battery when traveling the same distance at constant speed v on flat ground ($\theta = 0$, air drag and rolling resistance for the entire distance $2d$ and no regeneration) is $\boxed{E_f = 2dm(Rg + Cv^2)}$.

Comparison of Hilly Travel to Flat Travel

The energy expended traveling over the hill divided by the energy expended traveling on the flat is:

$$\boxed{r = \frac{E_h}{E_f} = \frac{Rg+Cv^2+(1-f)g\sin\theta}{2\left(Rg+Cv^2\right)}}.$$

This ratio will be greater than 1 when:

$Rg + Cv^2 + (1-f)g\sin\theta > 2(Rg + Cv^2)$ or

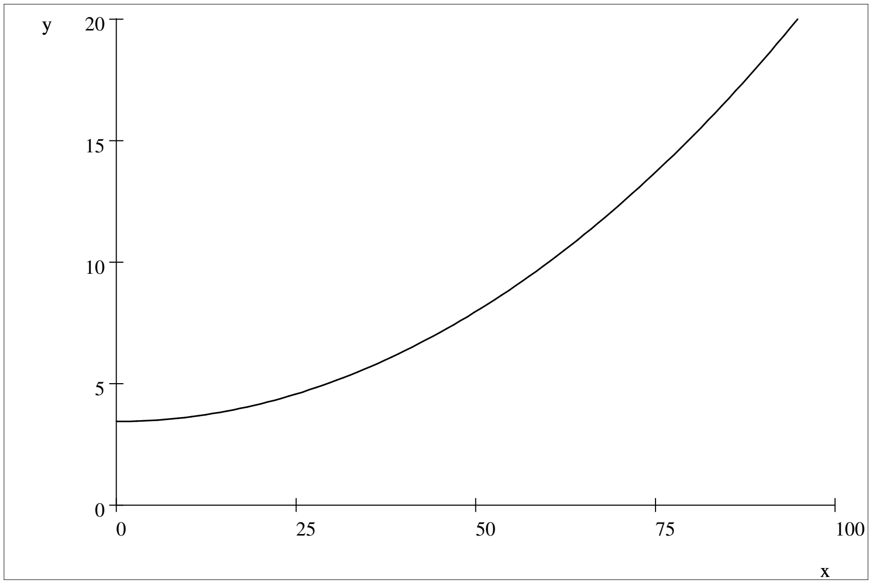
$(1-f)g\sin\theta > 2(Rg + Cv^2) - (Rg + Cv^2) = Rg + Cv^2$ or

$$\boxed{\sin\theta > \frac{Rg+Cv^2}{(1-f)g}} \text{ or } \boxed{v < \sqrt{\frac{g}{C}[(1-f)\sin\theta - R]}}$$

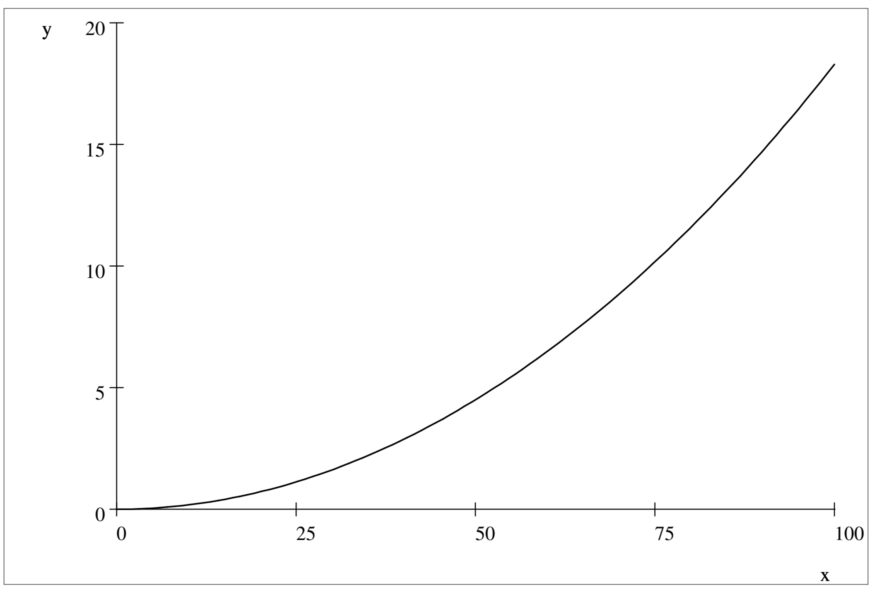
For the Nissan LEAF, $\boxed{f = 0.8,\ g = 9.8\frac{m}{s^2}, C = 3.08 \times 10^{-4}\frac{1}{m} \text{ and } R = 0.012}$.

Minimum Hill Angle vs Vehicle Speed and Regeneration

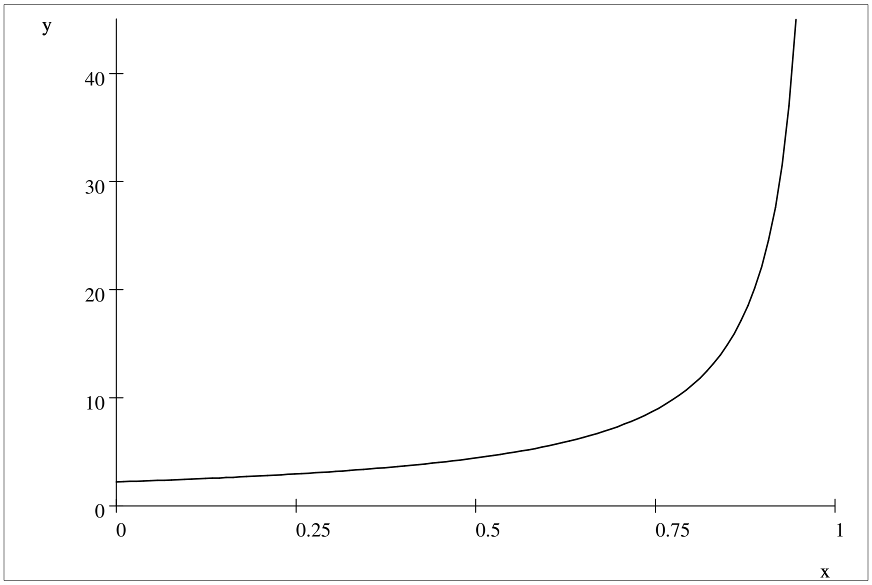
Here is a plot of the minimum hill angle (degrees) versus vehicle speed (mph) in order that hilly driving to be more efficient than flat driving:



The following graph shows the result if rolling resistance is neglected:

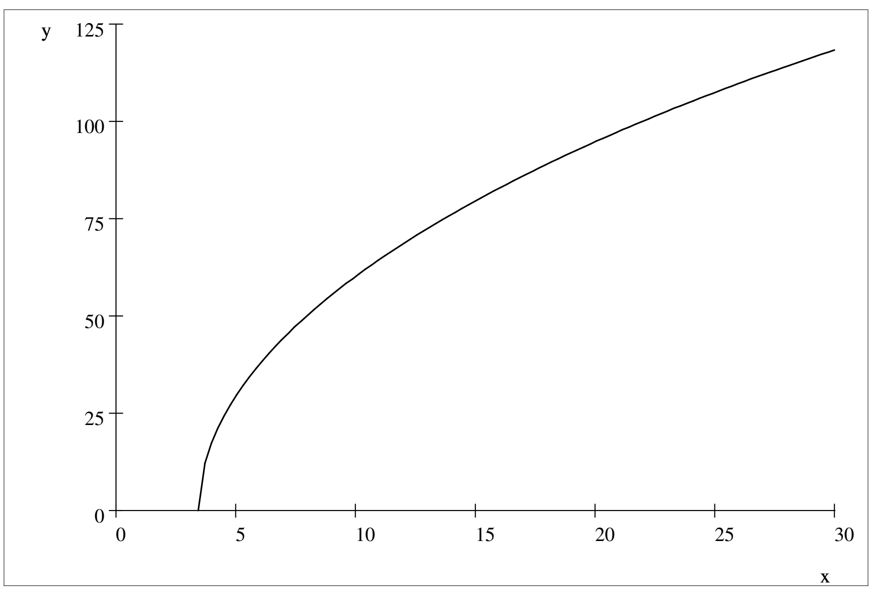


Here is a plot of the minimum hill angle (degrees) versus regeneration, f, going down the hill for 65 mph speed in order that hilly driving to be more efficient than flat driving:



Maximum Vehicle Speed vs Hill Angle and Regeneration

The following graph shows the maximum vehicle speed (mph) versus hill angle (degrees) in order that hilly driving is more efficient than flat driving:



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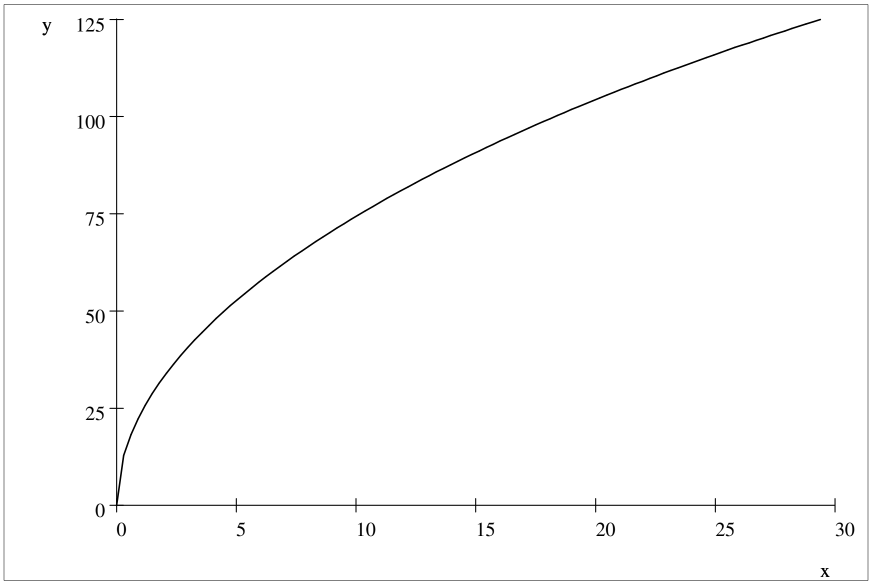
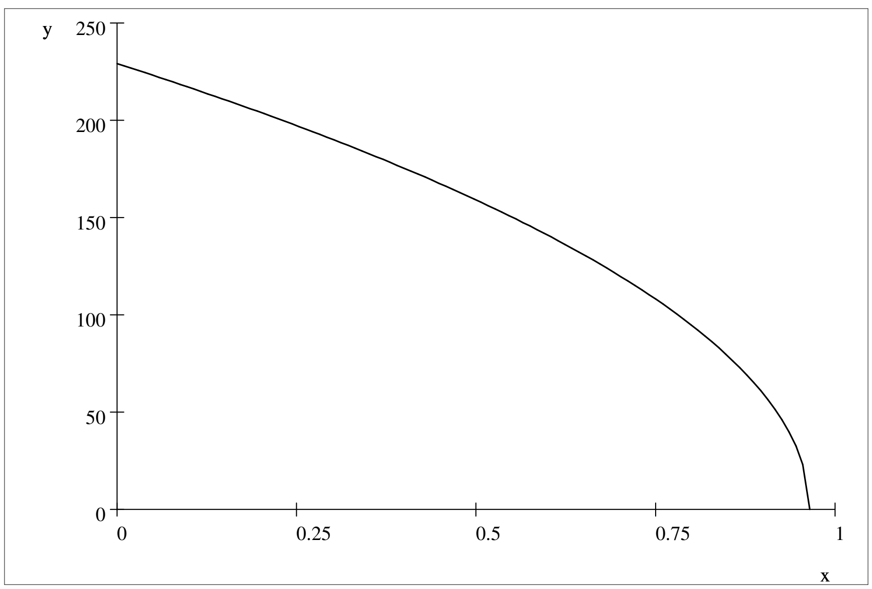


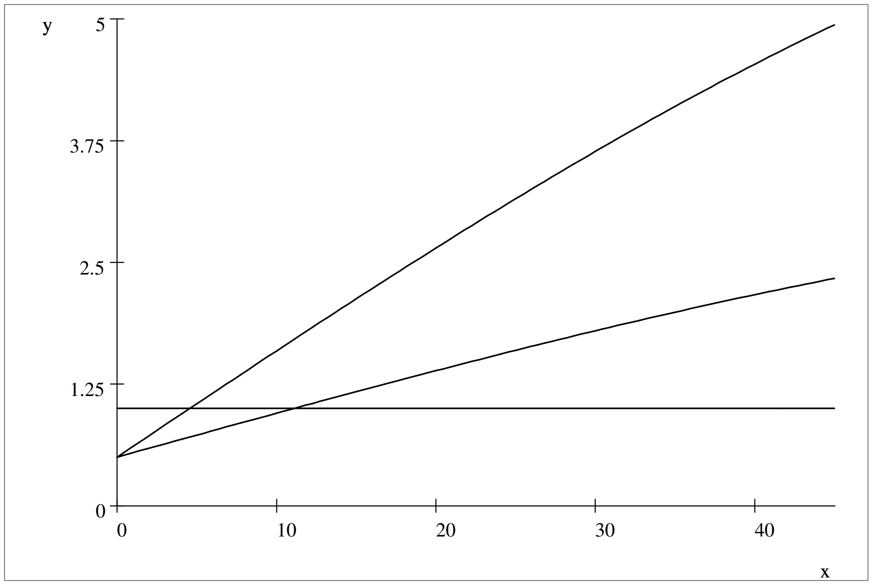
Here is a plot of the maximum vehicle speed (mph) versus fraction of regeneration, f, going down the hill for a hill angle of 20 degrees in order that hilly driving to be more efficient than flat driving:



The ratio of energy for hilly driving to flat driving for the Nissan LEAF is

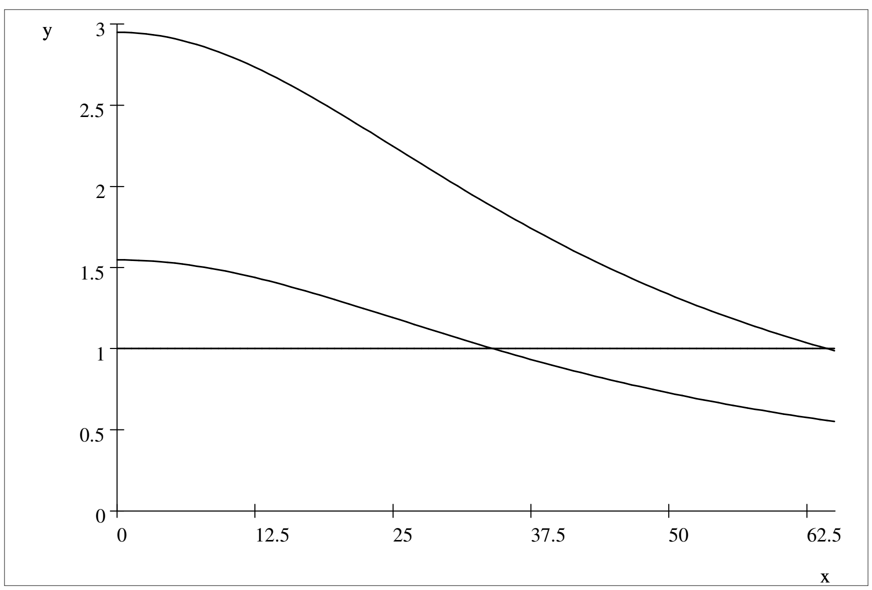
$$\boxed{r(f,\theta,v) \;=\; \frac{(1-0.8)9.8\sin\theta+0.012(9.8)+3.08\times10^{-4}v^2}{2\left(.012(9.8)+3.08\times10^{-4}v^2\right)}}.$$

Ratio versus hill angle for vehicle speeds 25 mph and 65 mph (lower curve):



Above the horizontal line for $r = 1$ hilly driving is more efficient than flat driving.

Ratio versus vehicle speed (mph) for hill angles 20 degrees and 10 degrees (lower curve):



Above the horizontal line for $r = 1$ hilly driving is more efficient than flat driving.

Conclusion

So, for the assumptions given in this article, an electric vehicle can use less battery energy going over a hill than it would use traveling on the flat for the same distance of travel if the hill angle is greater than a certain value for a given vehicle speed or the vehicle speed is less than a certain value for a given hill angle.

References

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