Duration of Shale-Gas Extraction in the United States

L. David Roper
http://arts.bev.net/roperldavid/
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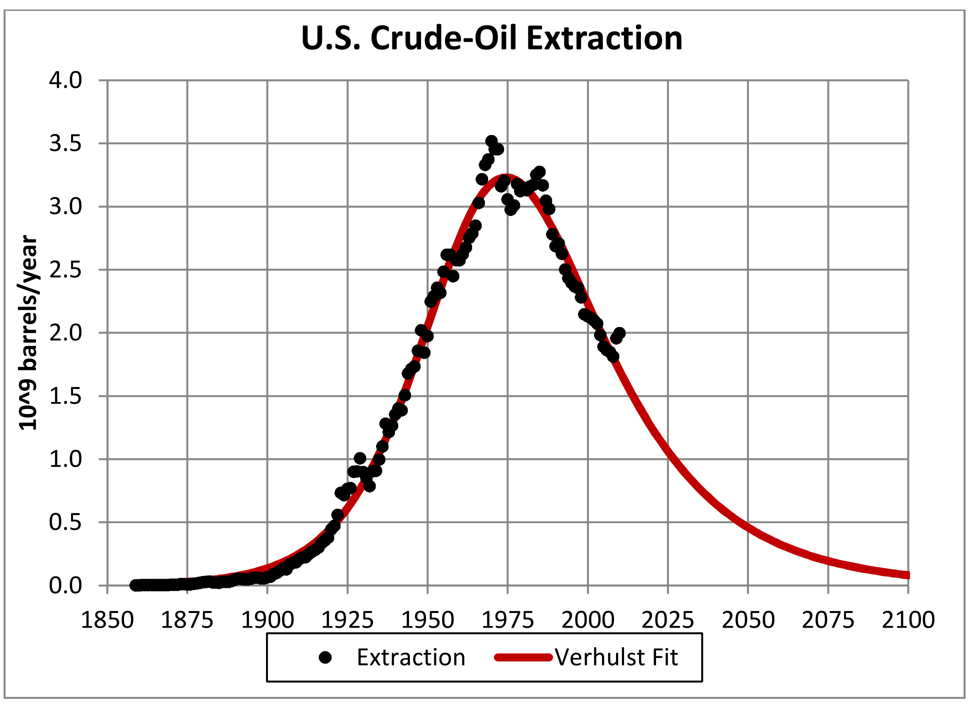
Introduction

One often reads and hears that there is enough shale gas in the United States ”to last 100 years”. That is almost a meaningless statement because extraction of a non-renewable natural resource follows a depletion curve that rises, usually exponentially, then slows down to a peak and then declines, usually exponentially. What is meant by “last” is not clear. It is shown in this article that words are not sufficient to describe the “duration” of a non-renewable natural resource; there is no substitute for doing the math.

Example of Crude-Oil Extraction in the United States

A good example is the extraction of crude oil in the United States:



The rise-exponential time constant is 15.7 years and the decline-exponential time constant is 1.84 times larger (28.9 years). [An interesting observation is the huge effort that was made to keep the peak from occurring and the huge effort that was made to keep the extraction from declining, both yielding small peaks around the overall peak. Another huge effort is now underway as the decline is about half way down.] The peak is at year 1974.5 and the total amount to be extracted is 253.8x10^9 barrels; that amount undoubtedly will change somewhat as future data become available.

Verhulst Function for Depletion Analyses

The depletion curve that fits the data in the above graph is the Verhulst function (http://www.roperld.com/science/minerals/VerhulstFunction.htm):

$$P(t) = \frac{Q_\infty}{n\tau} \frac{\left(2^n - 1\right)\exp\left(\frac{t - t_{\frac{1}{2}}}{\tau}\right)}{\left[1 + \left(2^n - 1\right)\exp\left(\frac{t - t_{\frac{1}{2}}}{\tau}\right)\right]^{\frac{n+1}{n}}}.$$

Q_∞ is the amount to be eventually extracted, τ is the rising exponential time constant, n τ is the declining exponential time constant and $t_{1/2}$ is the time at which the resource is one-half depleted. The maximum of P(t) occurs at $t_{\max} = t_{\frac{1}{2}} + \tau \ln\left(\frac{n}{2^n - 1}\right)$, which yields $P_{\max}\left(t_{\max}\right) = \frac{Q_\infty}{\tau} \frac{1}{\left(n+1\right)^{\frac{n+1}{n}}}$.

It is useful to define a "duration" for the extraction by the difference in the times when (f-1)/f has been extracted and when 1/f has been extracted:

$$D = \tau\left[\ln\left(\frac{f^n - 1}{2^n - 1}\right) - \ln\left(\frac{\left(\frac{f}{f-1}\right)^n - 1}{2^n - 1}\right)\right].$$

From that duration function one can calculate, setting f=10, the time interval between when one-tenth of the extraction occurs and when nine-tenths of it occurs (i.e., the middle 80% of the extraction amount); it is 90.5 years for crude-oil extraction for the United States. Call this the “**duration**” of crude-oil extraction for the United States. So, one could say, with this definition, that crude oil “will last” ~90 years in the United States.

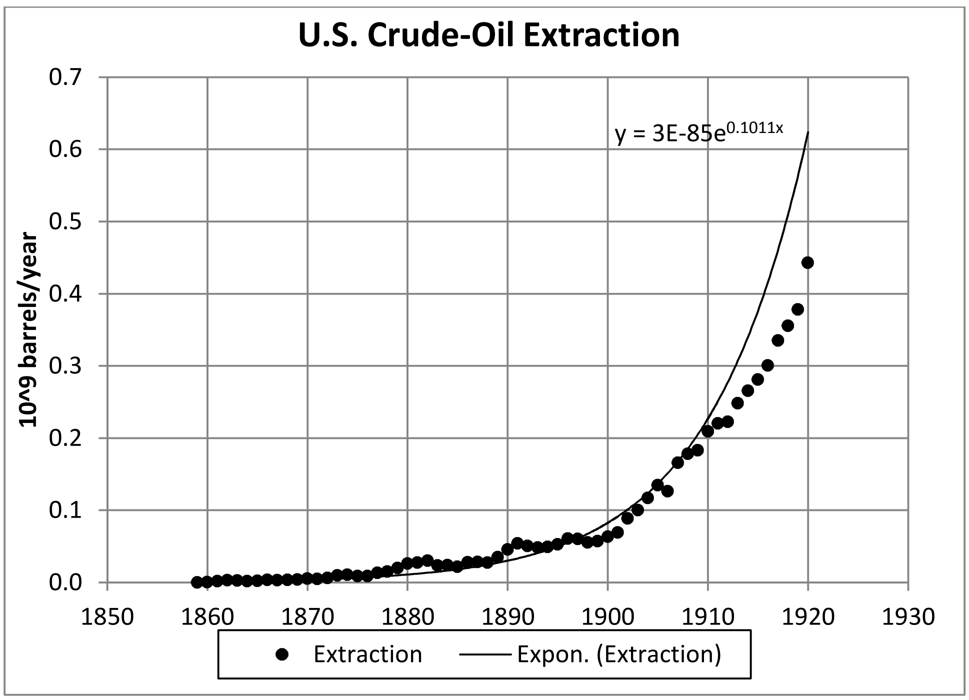
Shale-Gas Extraction in the United States

Shale-gas extraction in the United States has been ramping up very fast. Are there enough data now available to make predictions of when shale-gas extraction will peak and how long it will “last”?

Example of Crude-Oil Extraction in the United States

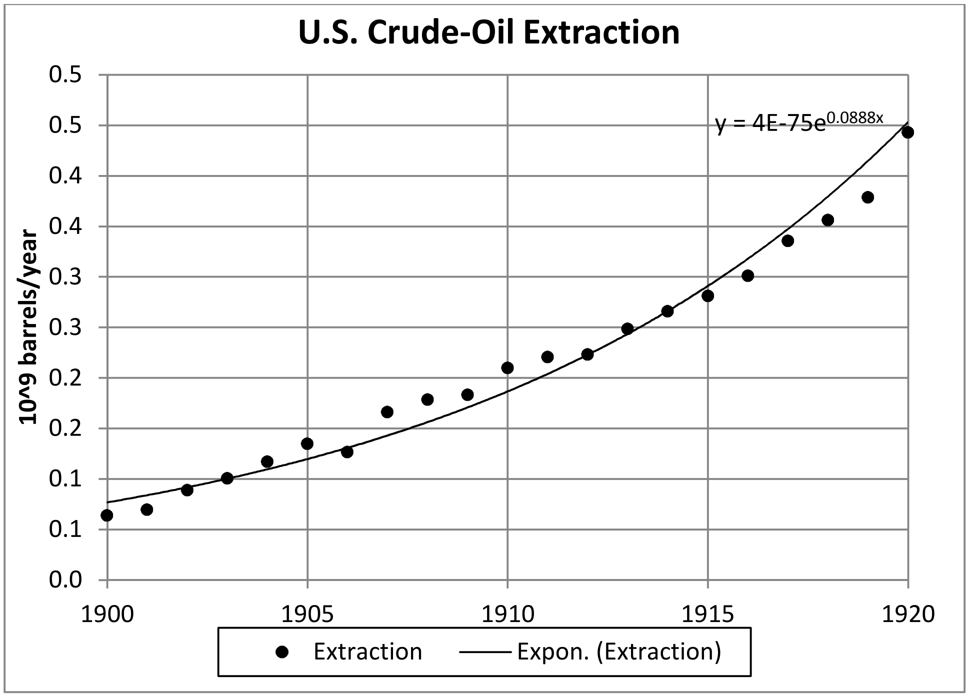
As an example, consider the years up to 1920 for crude-oil extraction in the United States. The following two graphs show two ways to determine the best rise exponential time constant for crude-oil extraction in the U.S.

In this graph all data from 1859 to 1920 are used:



The curve is an exponential fit to the data. The exponential time constant is 1/0.1011=9.89 years.

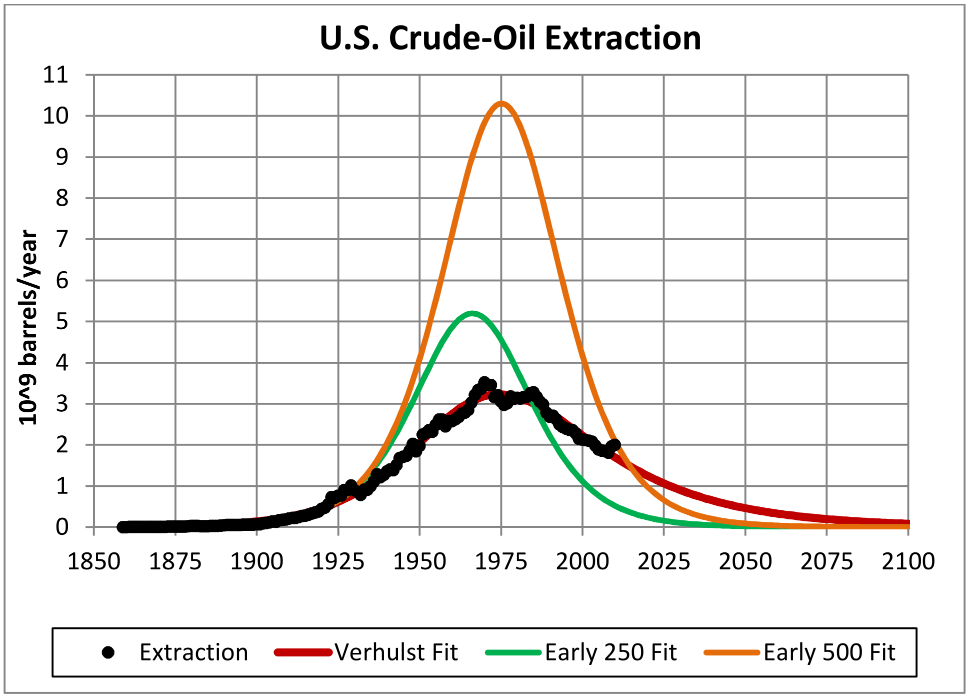
One could argue that only data since 1900 should be used, since it appears that a different rise rate occurs after then:



The curve is an exponential fit to the data. The exponential time constant is 1/0.0888=13.3 years.

Now, suppose that some reputable organization estimated in 1920 that the total amount to be extracted would be 250x10^9 barrels and another similar organization estimated that it might be twice that, 500x10^9 barrels.

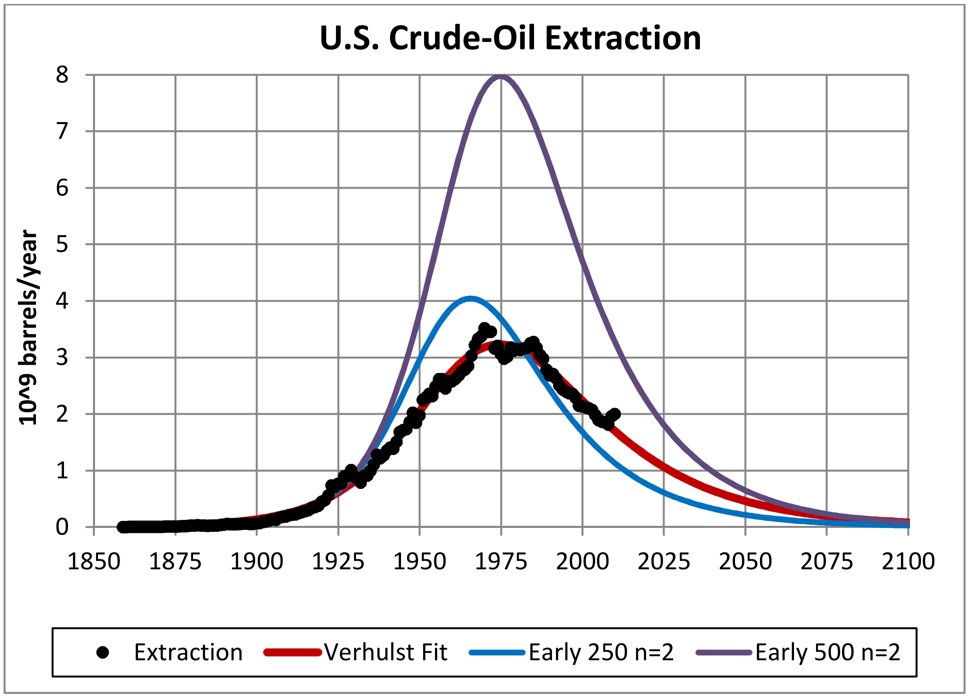
One can do a Verhulst-function fit to the data for both of these estimates to see when an extraction peak might occur. Since no information is available to determine the decline time constant, first assume that the depletion curve will be symmetric; i.e., the decline time constant will be the same as the rise time constant. Starting with the time constant 13.3 years and then allowing the peak year and the time constant to vary, one gets the following two fits to the early-time (to 1920) data for 250x10^9 barrels and 500x10^9 barrels eventual extraction:



The time constants, peak years and durations (see definition above) for the two early-time (to 1920) fits and the fit to all data up to 2010 are:

Amount to be extracted x10^9 barrels:	250	500	253.8 (Fit to 2010)
Time constant (years):	12.0	12.1	15.7 & 28.9
Peak year:	1966	1975	1979.8
Duration (years):	52.9	53.3	90.5

One could argue that the decline time constant should be greater than the rise time constant because of changing technologies to extract the harder-to-extract crude oil. If one sets the decline time constant to twice the rise time constant the fits to the early-time (to 1920) data are:



The time constants, peak years and durations (see definition above) for the two early-time (to 1920) fits and the fit to all data up to 2010 are:

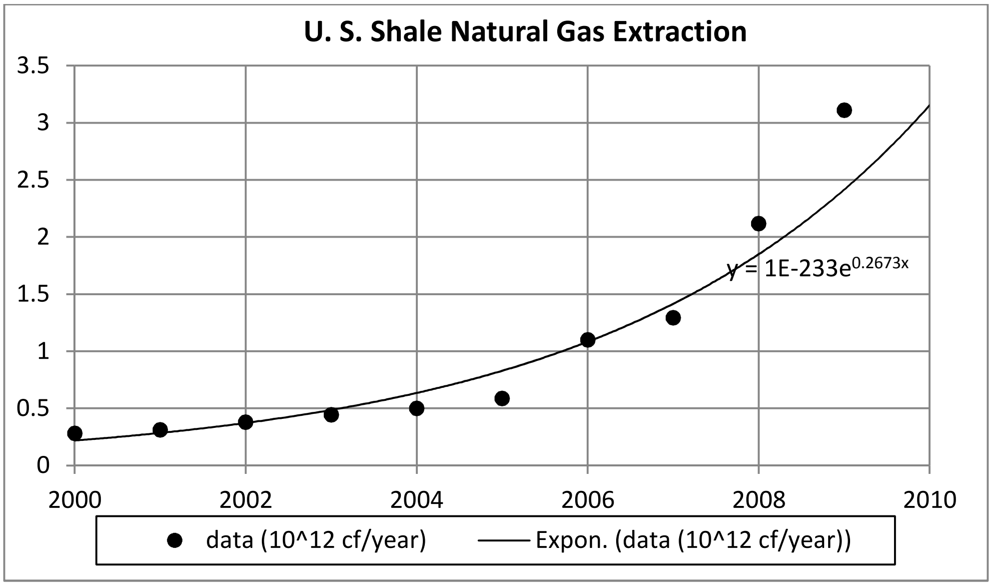
Amount to be extracted x10^9 barrels:	250	500	253.8 (Fit to 2010)
Time constants (years):	11.9 & 23.8	12.0 & 24.0	15.7 & 28.9
Peak year:	1966	1975	1979.8
Duration (years):	71.9	72.9	90.5

Note that the peak years do not change for the early-time fits, but the durations get closer to the actual duration when asymmetry is allowed.

This example of crude-oil extraction in the U.S. shows that analyzing early data to 1920 yields peak years nearly as large as the actual peak year and durations smaller than the actual duration.

Application of the Analysis to Shale-Gas Extraction in the United States

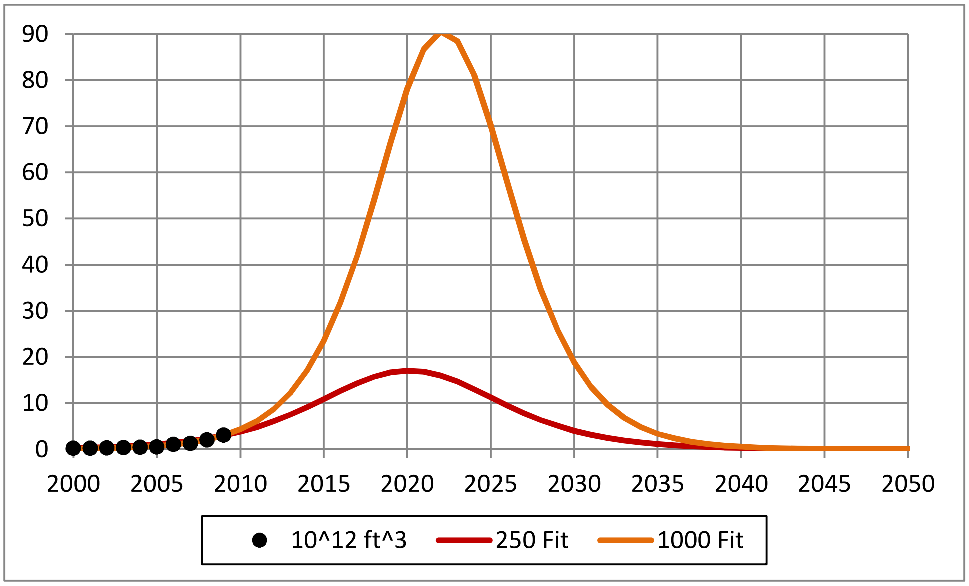
The graph below shows an exponential fit to the shale-gas extraction data to 2009:



The curve is an exponential fit to the data. The exponential time constant is 1/0.2673=3.74 years.

Various estimates of the amount of natural gas that can be extracted from shale range from 250x10^12 ft^2 to 1000x10^12 ft^2.

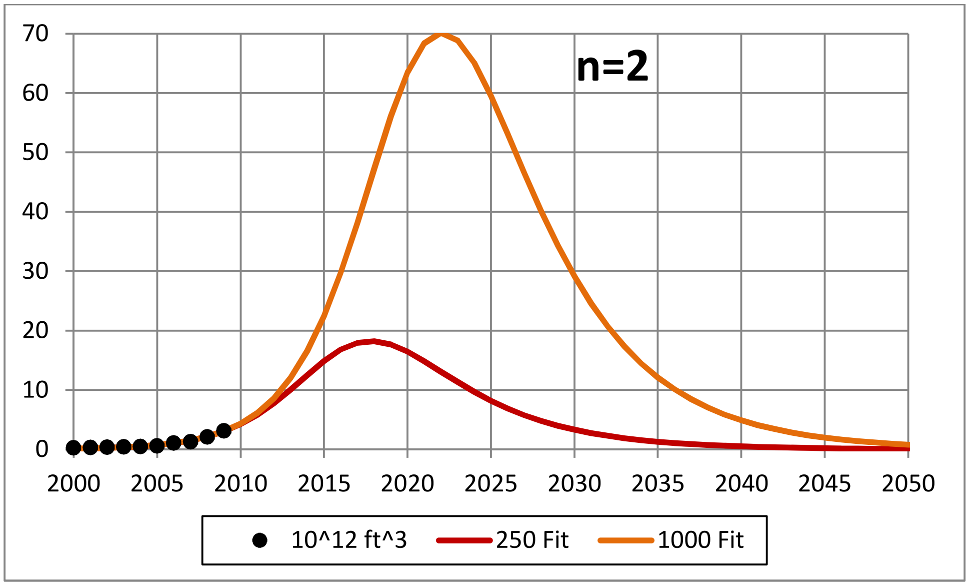
One can do a Verhulst-function fit to the data for both of these estimates' extremes to see when an extraction peak might occur. Since no information is available to determine the decline time constant, first assume that the depletion curve will be symmetric; i.e., the decline time constant will be the same as the rise time constant. Starting with the time constant 3.7 years and then allowing the peak year and the time constant to vary one gets the following two fits for eventual extraction of 250x10^12 ft^2 and 1000x10^12 ft^2:



The time constants, peak years and durations (see definition above) for the two fits are:

Amount to be extracted x10^12 ft^2:	250	1000
Time constant (years):	3.66	2.76
Peak year:	2020	2022
Duration (years):	16.1	12.1

One could argue that the decline time constant should be greater than the rise time constant because of changing technologies to extract the harder-to-extract natural gas from shale. If one sets the decline time constant to twice the rise time constant the fits are:

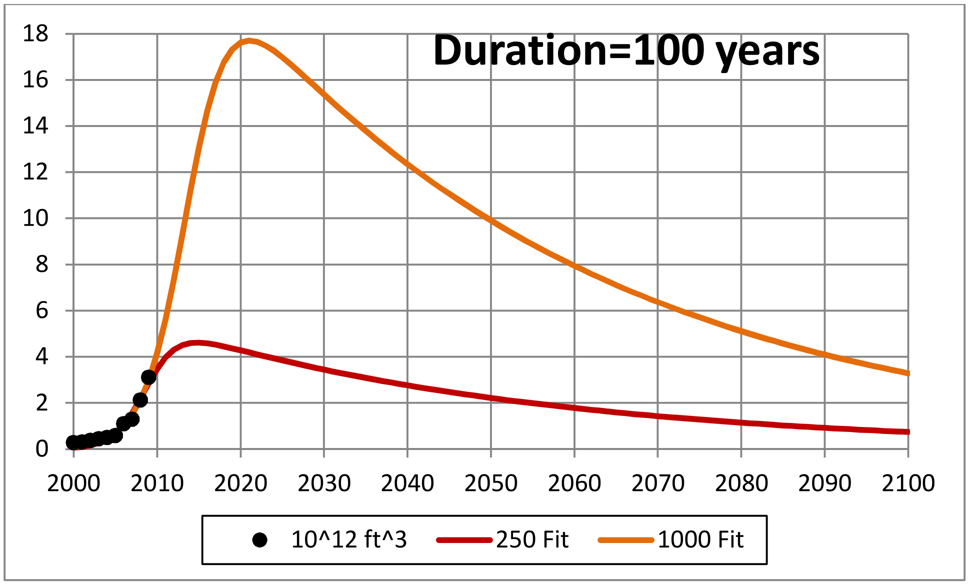


The time constants, peak years and durations (see definition above) for the two fits are:

Amount to be extracted x10^12 ft^2:	250	1000
Time constants (years):	2.64 & 5.27	2.74 & 5.49
Peak year:	2018	2022
Duration (years):	15.9	16.6

Note that the peak years do not change much for the fits when asymmetry is allowed. There is a difference of 4.5 years in the duration for the 1000 x10^12 ft^2 case. In all cases the duration is far less than 100 years.

One can fix the durations as defined above to be 100 years for the two fits and allow the other parameters to vary:



The time constants, peak years and durations (see definition above) for the these two fits are:

Amount to be extracted x10^12 ft^2:	250	1000
Time constants (years):	1.92 & 45.4	2.56 & 45.3
Peak year:	2014	2021
Duration (years):	100	100

In order to have 100 years for the duration, the peak value has to be greatly reduced. The locations of the peaks are still within a decade from 2010. Having such widely different rise and decline time constants seem highly unlikely.

Conclusion

To get the result that natural gas extracted from shale "will last 100 years", as claimed by many, the initial exponential rise has to slow down within the next decade and peak before or at about the end of the decade. It appears more likely that the "duration", as defined in this article, will be about 15 year instead of 100 years and that the extraction peak will occur within the next 15 years.

Works such as "will last 100 years" are not sufficient to describe the "duration" of a non-renewable natural resource, such as shale gas. There is no substitute for doing the math.

References

For more details about extraction of natural gas from shale, see
http://www.roperld.com/science/minerals/shalegas.htm