Pion-Nucleon P_{11} Partial Wave

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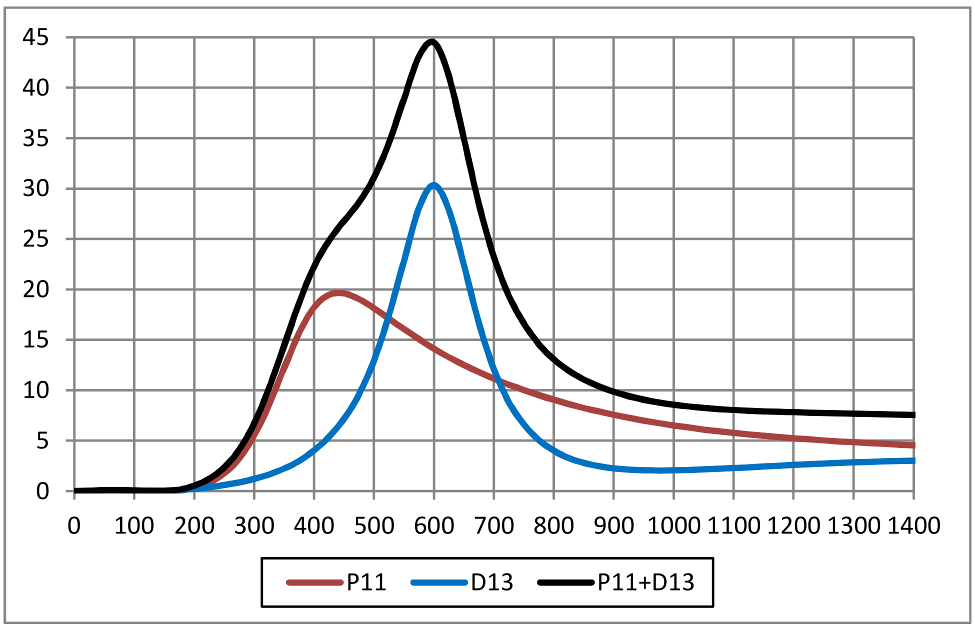
Introduction

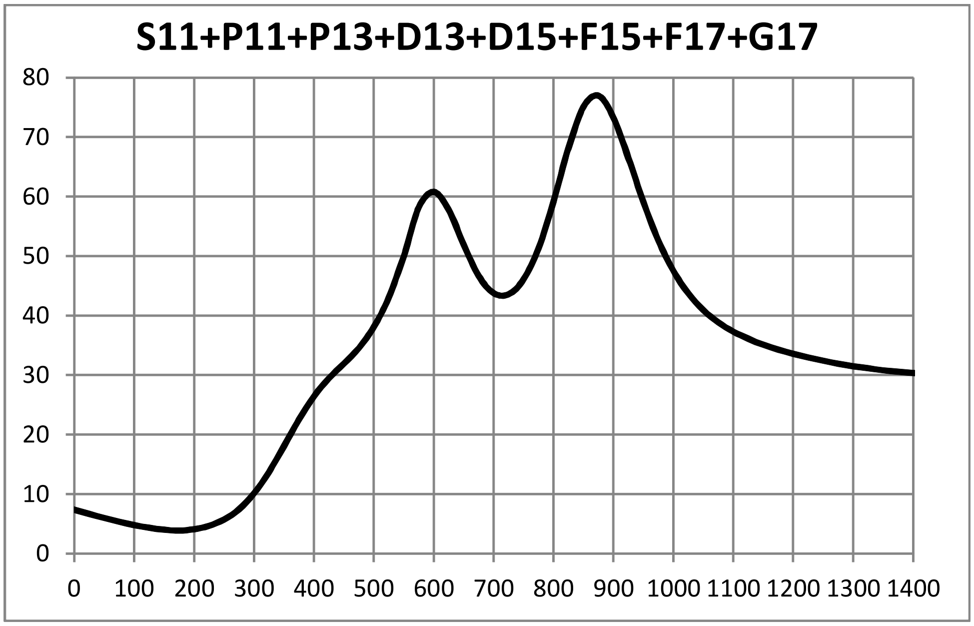
The author's PhD thesis at MIT in 1963 was a 0-700 MeV pion-nucleon partial-wave analysis[1]. A major conclusion[2] of that work was the existence of a new $\pi - N$ resonance at about 560 Mev pion laboratory kinetic energy, unofficially called the "Roper Resonance".

Since then pion-nucleon partial-wave analyses have been extended out to 3 GeV at George Washington University (GWU). This document will compare the original Roper analysis with the latest results and discuss a new parametrization of the P_{11} resonance by modified resonance equation only.

The P_{11} resonance was not found before my work because it was hidden in the $I = \frac{1}{2}$ total cross section under the 50% larger D13 resonance:



When all the $I = \frac{1}{2}$ partial-waves that are important below 1500 MeV lab kinetic energy are included the P_{11} resonance looks even less consequential in the total cross section:



Roper P_{11} Parametrization

The P_{11} parametrization used by Roper[1] is:

$$A = A_{res} + A_{nonres}$$

$$A_{res} = \frac{\Gamma_{el}}{2(q_r - q) - i(\Gamma_{el} + \Gamma_{in})}$$

$$\Gamma_{el} = \frac{4M(q - q_z)}{q(q + q_r)} \gamma_{el} \frac{(kr)^3}{1 + (kr)^2} \text{ (elastic resonance width)}$$

$$\Gamma_{in} = \gamma_{in}(k - k_{in})^3 \text{ for } k \geqslant k_{in} \text{ (inelastic resonance width)}$$

$$q = \sqrt{1 + k^2} \text{ (c.m. pion energy)}$$

$$k = M\sqrt{\frac{E(E + 2M)}{(M+1)^2 2ME}} \text{ (c.m. pion momentum)}$$

$$A_{nonres} = \frac{1}{2i}[\eta_{nr} \exp(2\delta_{nr}) - 1]$$

$$\delta_{nr} = a_1 k^3 + a_2 k^4 + a_3 k^5 + a_4 k^6$$

$$\eta_{in} = \exp(-2\nu)$$

$$\nu = b_1(k - k_{in})^3 + b_2^4(k - k_{in}) + b_3^5(k - k_{in}) + b_4^6(k - k_{in}) \text{ for } k \geqslant k_{in}$$

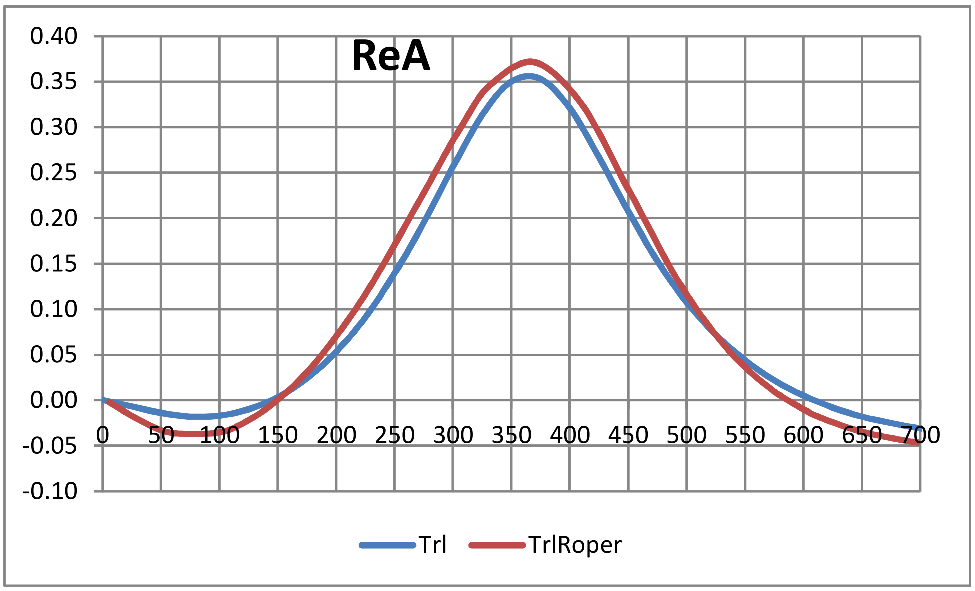
M = proton mass = (6.723) and E = pion laboratory kinetic energy are in units of the pion mass. The interaction range, r, was fixed at 0.71 (1 fermi).

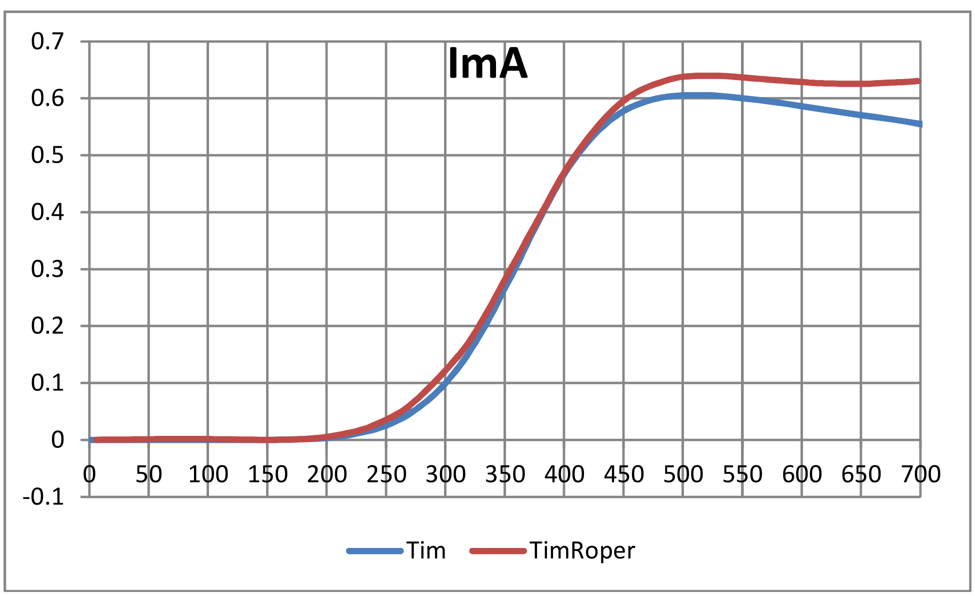
The parameters of the fit are:

$$q_r = 3.3378$$
$$q_z = 1.736$$
$$k_{in} = 1.479$$
$$\gamma_{el} = 0.4897$$
$$\gamma_{in} = 0.3885$$
$$a_1 = -2.181 \times 10^{-8}, a_2 = -6.449 \times 10^{-8}, a_3 = -1.861 \times 10^{-10}, a_4 = 1.272 \times 10^{-9}$$
$$a_1 = 3.8733 \times 10^{-2}, a_2 = -3.904 \times 10^{-3}, a_3 = -2.724 \times 10^{-3}, a_4 = 2.320 \times 10^{-3}$$

Roper Fit

The following graph compares the 1965 Roper[1] fit to the recent results at GWU:

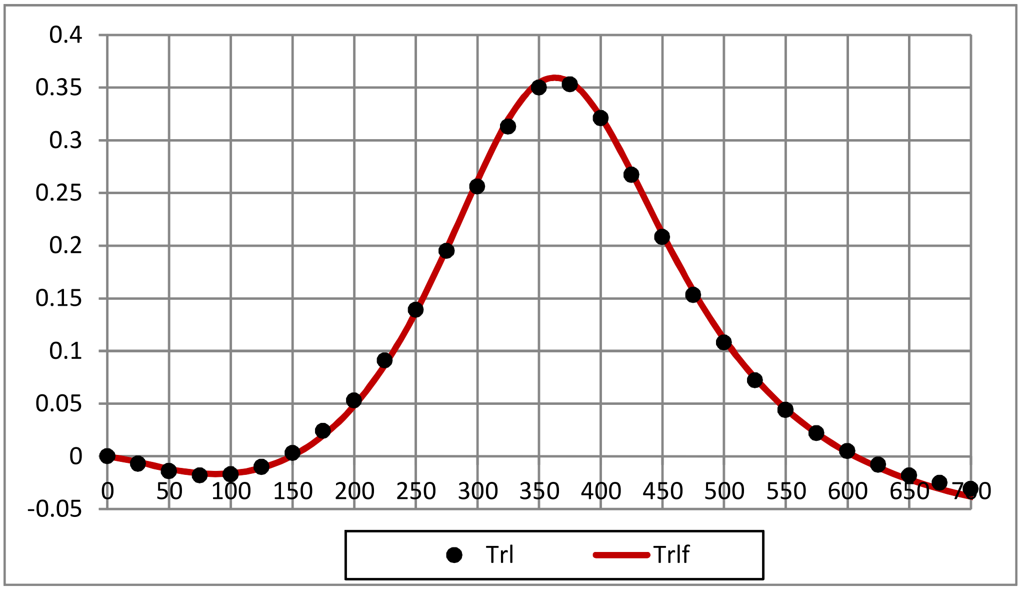


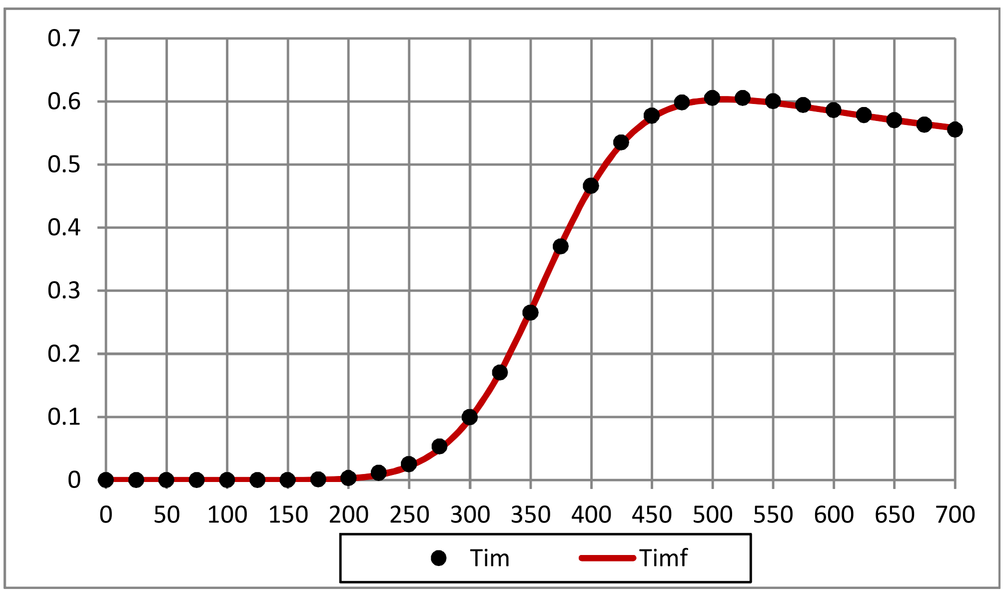


This is not too bad for 45 years ago!

Better Parametrization

I redid the parametrization as follows: $\Gamma_{in} = \gamma_{in} \frac{[r_{in}(k-k_{in})]^3}{1+[r_{in}(k-k_{in})]^2}$ for $k \geqslant k_{in}$ to get better behavior of the inelastic width at high energies. With this change I was able to fit the recent P11 results from 0 to 700 MeV without any background:





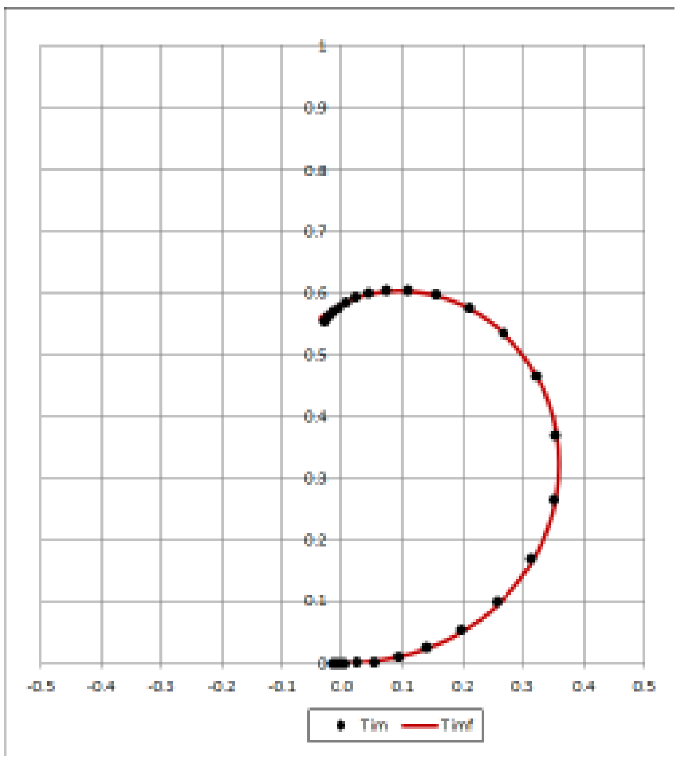
This is an excellent fit!

The parameters of the fit are:

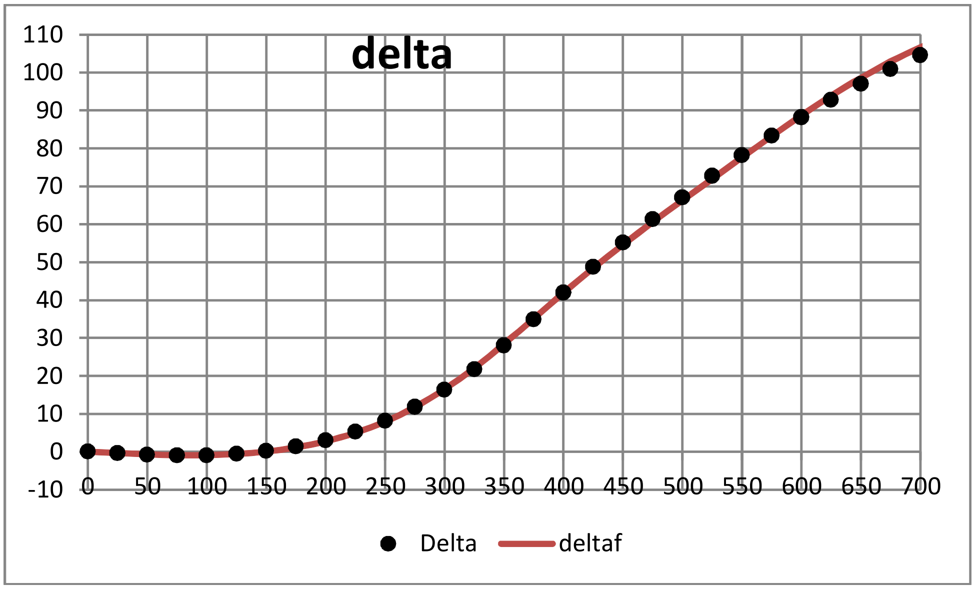
$$q_r = 3.398$$
$$q_z = 1.731$$
$$k_{in} = 1.479$$
$$\gamma_{el} = 2346$$
$$\gamma_{in} = 20.50$$
$$r = 0.02805$$
$$r_{in} = 0.3027$$

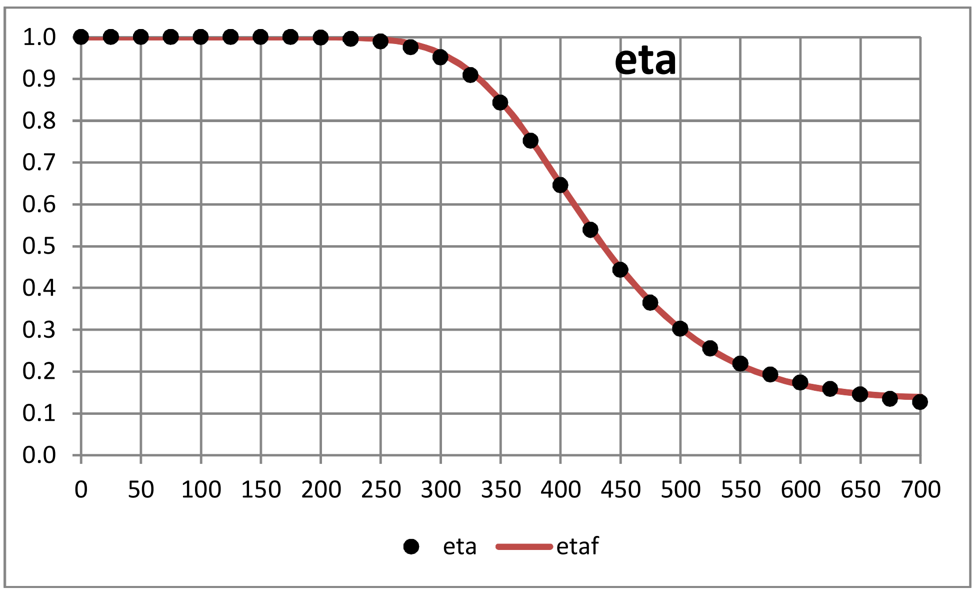
The number of variable parameters is 7 instead of 13 as in the 1965 work. Apparently the added zero factor, $\frac{q-q_z}{q}$ provides enough "background" to the Layson resonance formula[3].

The Argand diagram of the partial-wave amplitude is:



The phase shift, δ, and the absorption parameter, η :





Total Energy (W) Parametrization

I redid the "better" parametrization above to make the resonance energy the total c.m. energy, W , instead of the c.m. energy, q .

The parameters of the fit are:

$$W_r = 10.88$$
$$W_z = 8.532$$
$$k_{in} = 1.351$$
$$\gamma_{el} = 73.52$$
$$\gamma_{in} = 3852$$
$$r = 0.2298$$
$$r_{in} = 0.05235$$

for the parametrization

$$A = \frac{\Gamma_{el}}{2(W_r - W) - i(\Gamma_{el} + \Gamma_{in})}$$

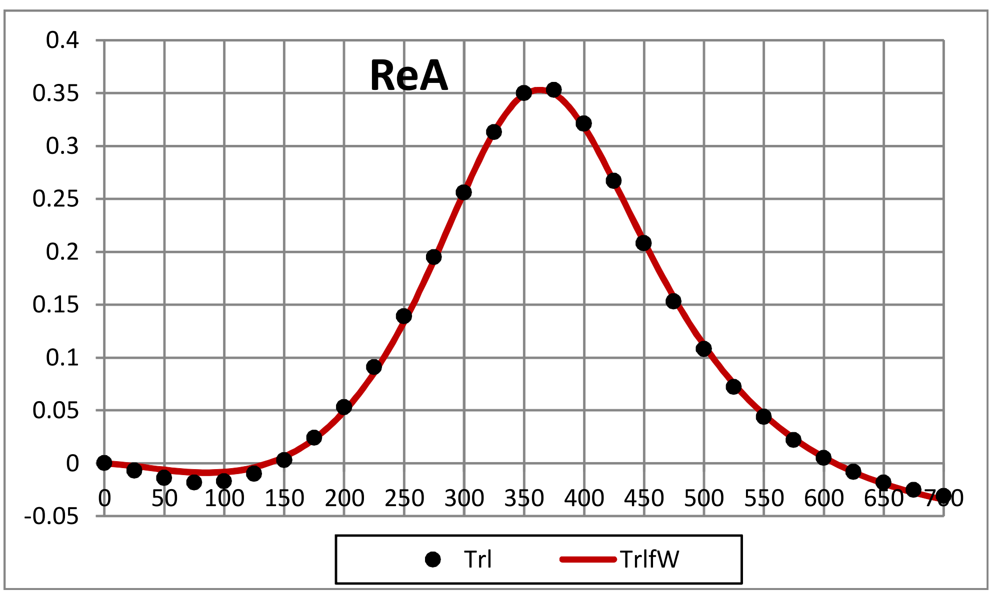
$$\Gamma_{el} = \frac{4M(W - W_z)}{W(W + W_r)} \gamma_{el} \frac{(kr)^3}{1 + (kr)^2}$$ (elastic resonance width)

$$\Gamma_{in} = \gamma_{in} \frac{[r_{in}(k - k_{in})]^3}{1 + [r_{in}(k - k_{in})]^2}$$ for $k \geq k_{in}$ (inelastic resonance width).

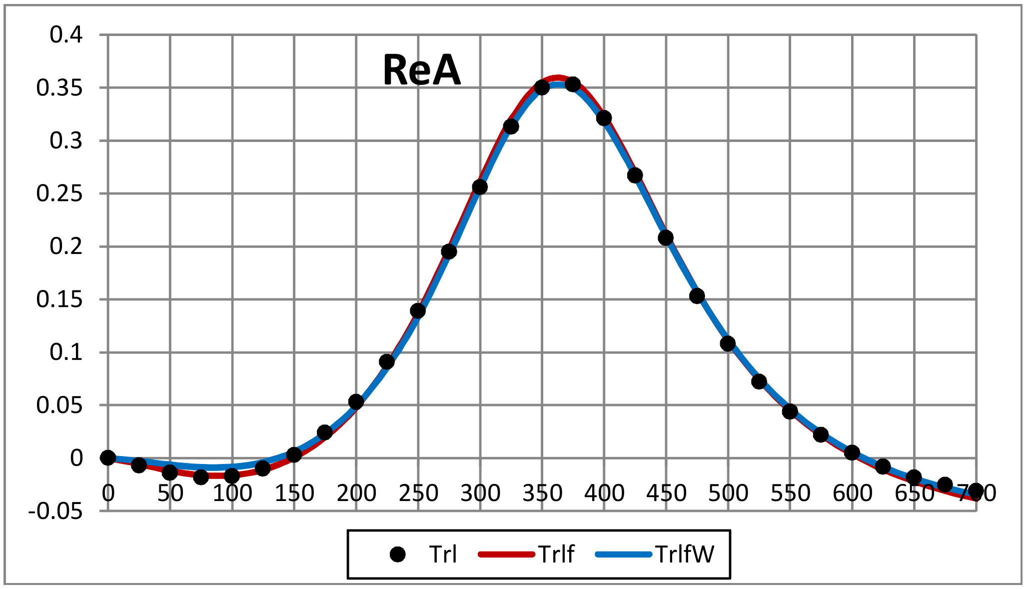
In terms of the total energy W:

$$k = \frac{1}{2W} \sqrt{\left[W^2 - (M+1)^2\right]\left[W^2 - (M-1)^2\right]}.$$

The chi square for this fit is slightly larger than for the q parametrization. The only visible difference in the plots is for ReA :



The W parametrization does not fit the energies below the zero as well as does the q parametrization, although it fits the peak slightly better as shown in the following graph:



Extension of Total-Energy Parametrization to 3000 MeV Lab Energy

A higher energy resonance was added in an attempt to fit the lab energy range 0 to 3000 MeV. The parametrization is:

$$A = \frac{\Gamma_{el}}{2(W_r - W) - i(\Gamma_{el} + \Gamma_{in})} + \frac{\Gamma_{2el}}{2(W_{2r} - W) - i(\Gamma_{2el} + \Gamma_{2in})}$$

$$\Gamma_{el} = \frac{4M(W - W_z)}{W(W + W_r)} \gamma_{el} \frac{(kr)^3}{1 + (kr)^2} \text{ (elastic resonance width)}$$

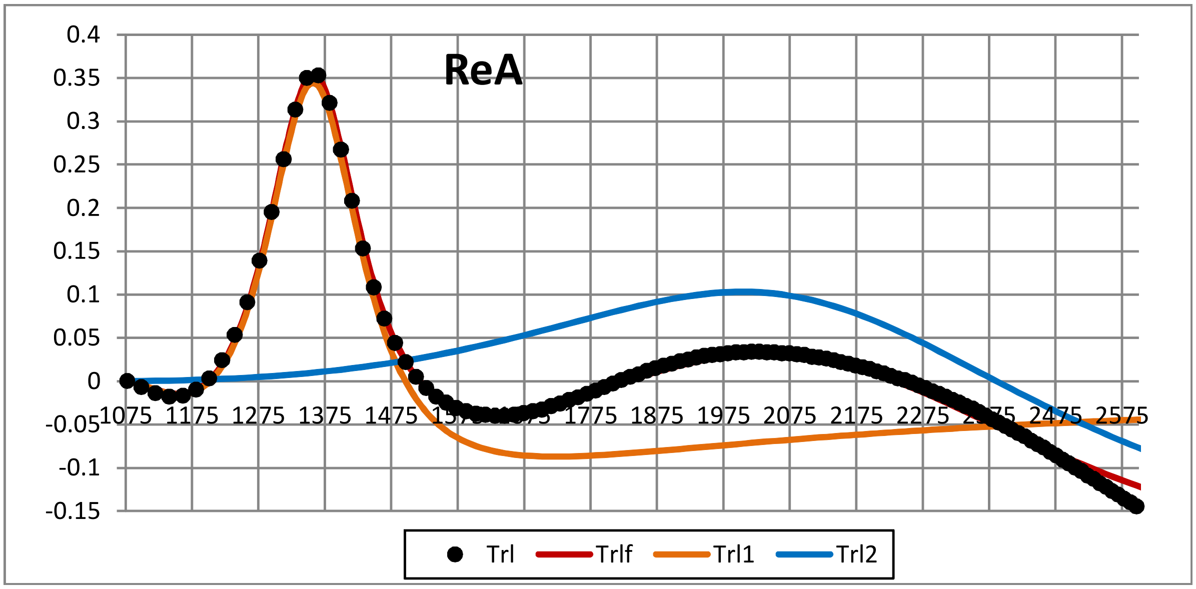
$$\Gamma_{in} = \frac{4M}{W + W_r} \gamma_{in} \frac{[r_{in}(k - k_{in})]^3}{1 + [r_{in}(k - k_{in})]^2} \text{ for } k \geq k_{in} \text{ (inelastic resonance width)}$$

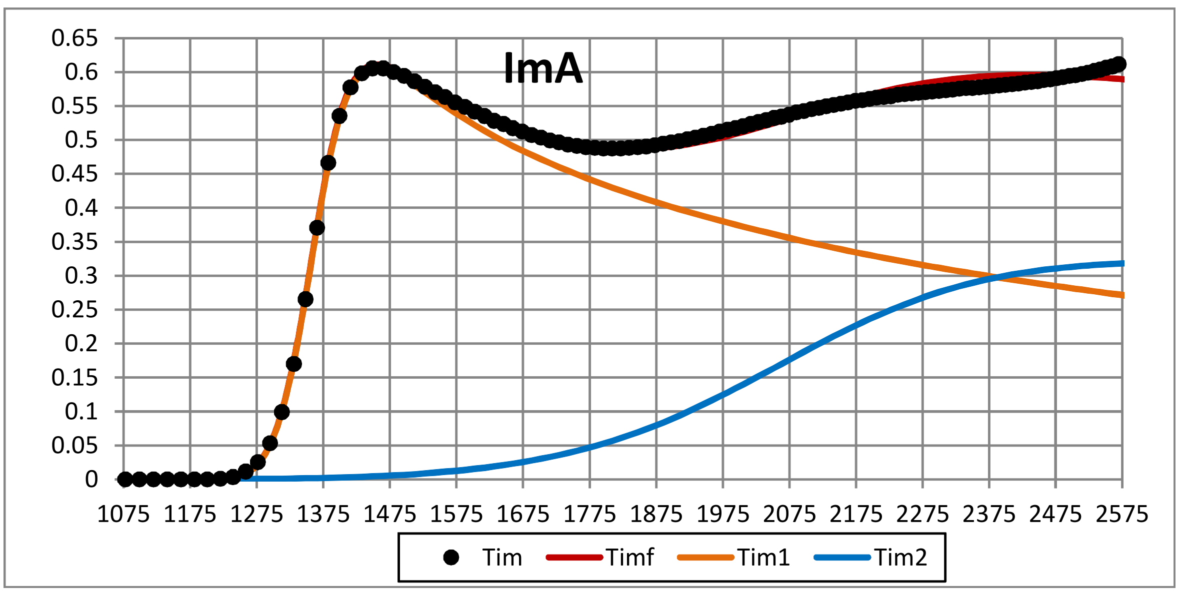
$$\Gamma_{2el} = \frac{4M}{W + W_r} \gamma_{2el} \frac{(kr_2)^3}{1 + (kr_2)^2}$$

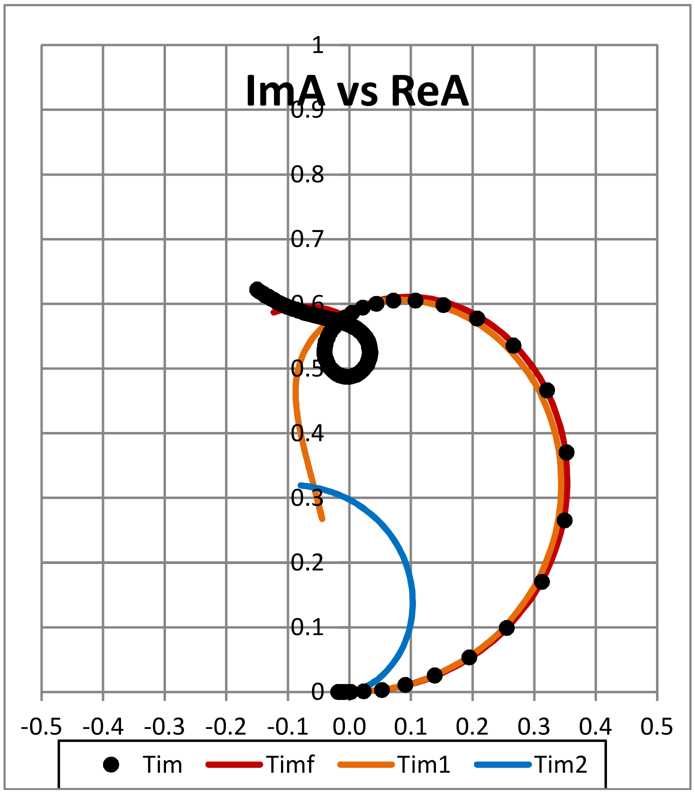
$$\Gamma_{2in} = \frac{4M}{W + W_r} \gamma_{2in} \frac{[r_{in}(k - k_{in})]^3}{1 + [r_{in}(k - k_{in})]^2} \text{ for } k \geq k_{2in}$$

Note that the Layson factor[3], $\frac{4M}{W+W_r}$, has been inserted into the equations for Γ_{in} and Γ_{2in}; that only slightly improved the fit.

The fit is:


ReA
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
1075 1175 1275 1375 1475 1775 1875 1975 2075 2175 2275 2475 2575
Trl Trlf Trl1 Trl2


ImA
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1075 1175 1275 1375 1475 1575 1675 1775 1875 1975 2075 2175 2275 2375 2475 2575
Tim Timf Tim1 Tim2



The parameters of the fit are:

$$\begin{array}{ll} W_r = 10.69 & W_{2r} = 17.16 \\ W_z = 8.602 & - \\ k_{in} = 1.110 & k_{2in} = 0 \\ \gamma_{el} = 14.40 & \gamma_{2el} = 19730 \\ \gamma_{in} = 628.9 & \gamma_{2in} = 0.01201 \\ r = 0.4434 & r_2 = 0.008747 \\ r_{in} = 0.07630 & r_{2in} = 95.37 \end{array}$$

Note that adding the second resonance caused the low-energy behavior to be better fitted.

Appendix: Some Kinematics

The final c.m. inelastic momentum, k' , is used in the parametrizations given above. The equation for the incident c.m. momentum, in units of the incident pion mass, is

$$k = M\sqrt{\frac{E(E+2)}{(M+1)^2+2ME}} = \frac{1}{2W}\sqrt{\left[W^2-(M+1)^2\right]\left[W^2-(M-1)^2\right]}$$

where E is the incident-pion lab kinetic energy, W is the total c.m. energy and M is the target nucleon mass, all in units of the incident pion mass. If the final inelastic state has total mass M' the threshold incident-pion lab kinetic energy is

$$E = \frac{M'^2 - (M+1)^2}{2M}.$$

The threshold incident c.m. momentum is

$$k = \frac{1}{2M'}\sqrt{\left[M'^2 - (M+1)^2\right]\left[M'^2 - (M+1)^2 + 4M\right]}.$$

If $M' = M + m$, a two-body final state that contains the nucleon and some meson, then

$$m = -M + \sqrt{(M+1)^2 - 2(M-k^2) + 2\sqrt{(M-k^2)^2 + k^2(M+1)^2}}\ .$$

Theoretical Attempts to Explain the P11 resonance

There have been attempts using about every possible theoretical scheme in particle physics to explain the existence of the P11 resonance.

- http://arxiv.org/PS_cache/arxiv/pdf/1001/1001.5080v1.pdf ; **Study of nucleon resonances at EBAC@JLab**: Abstract. We present the dynamical origin of the P_{11} nucleon resonances resulting from a dynamical coupled-channels (DCC) analysis of meson production reactions off a nucleon target, which is conducted at Excited Baryon Analysis Center (EBAC) of Jefferson Lab. Two resonance poles are found in the energy region where the Roper resonance P_{11}(1440) was identified. Furthermore, the two resonance poles and the next higher resonance pole corresponding to P_{11}(1710) are found to originate from a single bare state.
- http://www.dgip.utfsm.cl/publico/Pub_aca/fis94.pdf ; **Electroexcitation of the Roper resonance for** $1.7 < Q^2 < 4.5\ GeV^2$ **in** $\vec{e}p \to en\pi^+$: Available model predictions for $\gamma^* p \to N(1440)P_{11}$ allow us to conclude that these results provide strong evidence in favor of $N(1440)P_{11}$ as a first radial excitation of the $3q$ ground state. The results of the present paper also confirm the conclusion of our previous analysis for $Q^2 < GeV^2$ that the presentation of $N(1440)P_{11}$ as a q^3G hybrid state is ruled out.
- http://arxiv.org/PS_cache/arxiv/pdf/0803/0803.3020v1.pdf ; **First Lattice Study of the** $N - P_{11}(1440)$ **Transition Form Factors**: In this work, we attempt to use first-principles lattice QCD (for the first time) to provide a model-independent study of the Roper-nucleon transition form factor. Conclusion and Outlook: This exploratory study demonstrates that the nucleon-Roper transition form factors can be measured from a first-principles lattice calculation. Using the fitting approach with appropriately chosen operator smearing, we not only improve the signal in the nucleon-nucleon form factors (especially at large momenta), but also successfully extract the nucleon-Roper. We may vary the projector used in the threepoint function to further improve the signal. In the future, since the pion mass in our simulation is very heavy at 720 MeV, we would like to consider lighter pion masses as well as starting work on unquenched anisotropic lattices.
- http://arxiv.org/PS_cache/hep-ph/pdf/0306/0306199v3.pdf ; **Roper Resonance and** $S_{11}(1535)$ **from Lattice QC**D: Using the constrained curve fitting method and overlap fermions with the lowest pion mass at 180 MeV, we observe that the masses of the first positive and negative parity excited states of the nucleon tend to cross over as the quark masses are taken to the chiral limit. Both results at the physical pion mass agree with the experimental values of the Roper resonance ($N^{1/2+}(1440)$) and S_{11} ($N^{1/2-}(1535)$). This is seen for the first time in a lattice QCD calculation.

- http://juwel.fz-juelich.de:8080/dspace/bitstream/2128/2384/1/24867.pdf ; **What is the structure of the Roper Resonance**: We investigate the structure of the nucleon resonance $N^*(1440)$ (Roper) within a coupled-channel meson exchange model for pion-nucleon scattering. The coupling to $\pi\pi N$ states is realized effectively by the coupling to the σN, $\pi\Delta$, an ρN channels. The interaction within and between these channels is derived from an effective Lagrangian based on a chirally symmetric Lagrangian, which is supplemented by well known terms for the coupling of the Δ isobar, the ω meson, and the "σ," which is the name given here to the strong correlation of two pions in the scalar-isoscalar channel. In this model the Roper resonance can be described by meson-baryon dynamics alone; no genuine $N^*(1440)$ (three quark) resonance is needed in order to fit πN phase shifts and inelasticities.
- http://th-www.if.uj.edu.pl/~acta/vol29/pdf/v29p2397.pdf **The Dynamics of πN Scattering and the Baryon Spectrum**: In the present analysis I will detail a procedure for calculating the baryon spectrum as a solution of an eigenvalue problem that generates both the mass and width of the state. This is illustrated for the case of the Δ and Roper resonances. ...a valence quark model for the baryon with meson coupling to the quarks and predict the baryon spectrum as resonances that decay to observed mesons and baryons. The width of these states is determined by the degree of meson dressing included.
- http://www.slac.stanford.edu/econf/C070910/PDF/166.pdf **Partial-Wave Analysis and Spectroscopy from πN Scattering to Pion-Electroproduction**: The prominent $N(1440)P_{11}$ resonance is clearly evident in both KH and GW/VPI analyses..., but occurs very near the ΔN, ηN and ρN thresholds, making a BW fit questionable. The N(1440) is the single resonance which manifests itself through two poles on different Riemann sheets(with respect to the $\pi\Delta$-cut). Due to the nearby $\pi\Delta$ threshold, both P_{11} poles are not far from physical region. There is a shift between pole positions at two sheets, due to a non-zero jump at the $\pi\Delta$-cut. Our conclusion is that a simple BW parametrization cannot account for such complicated structure.
- http://www.slac.stanford.edu/econf/C070910/PDF/inoue.pdf ; **Pion-Nucleon P_{33} and P_{11} Scatterings in the Lippmann-Schwinger Approach**: We turn to P_{11} partial wave scattering. Fig.3 left shows the result in the model with a baryon pole in addition to nucleon one. Entire feature of data is well reproduced by putting two baryons at 939[MeV] and 1410[MeV]. .... Note that the bare energy of the baryon is 1410[MeV], and the resonance appears at energy slightly above. This means that the baryon is shifted to upward. This result doesn't help the conventional quark model at all. We know that the model does not predict a positive parity excited nucleon around the energy, but around 1550[MeV]. Hence, it must be shifted to downward largely in order to reproduce data. However, the present result shows that a baryon state will be shifted to upward by coupling to πN scattering state. After all, the puzzle of Roper resonance in the model, still remains.
- http://arxiv.org/PS_cache/arxiv/pdf/1008/1008.0214v1.pdf ; **Extraction of P_{11} Resonance from πN Data and Its Stability**: Abstract. An important question about resonance extraction is how much resonance poles and residues extracted from data depend on a model used for the extraction, and on the precision of data. We address this question with the dynamical coupled-channel (DCC) model developed in Excited Baryon Analysis Center (EBAC) at JLab. We focus on the

P_{11} πN scattering. We examine the model-dependence of the poles by varying parameters to a large extent within the EBAC-DCC model. We find that two poles associated with the Roper resonance are fairly stable against the variation. We also develop a model with a bare nucleon, thereby examining the stability of the Roper poles against different analytic structure of the P_{11} amplitude below πN threshold. We again find a good stability of the Roper poles.

- http://theory.tifr.res.in/strong2010/program/Talks/Session1/D_Richards.pdf : Roper (1440): lightest positive parity excitation of the nucleon – lighter than the N(1535) negative-parity excitation. Hard to reconcile with constituent quark model. Single bare state in P11 channel gives rise to three poles: two around the Roper N*(1440), and the other around the N*(1710).

References

[1] L. D. Roper, R. M. Wright, and B. T. Feld, Phys. Rev. 138, B190 (1965)

[2] L. D. Roper, Phys. Rev. Letters 12, 340 (1964)

[3] W. M. Layson, Nuovo Cimento 27, 724 (1963)