Solar Collectors Orientation

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Orientation

Solar thermal or photovoltaic collectors need to be installed at an angle relative to the Earth surface to maximize the amount of solar energy collected at specified times of the year.

Let φ=latitude & θ=angle of solar collector relative to the Earth surface.

Only northern latitudes are considered here.

When the collector angle θ=0° the angle β of the Sun's rays relative to the perpendicular to the Earth surface for the season points of the year are

March equinox:	φ
June summer:	φ-23.5°
September equinox:	φ
December winter:	φ+23.5°

One can linearly interpolate between those four months to get β for the other eight months. (This procedure is not accurate because months have different numbers of days and the season points occur after the middle of four months, but it is accurate enough for orienting solar collectors.)

For an arbitrary collector angle θ the angle β(month) = θ – (month angle for θ = 0°).

For example, for Blacksburg VA at latitude φ = ~37° for several values of the collector angle θ, the month angles β are approximately:

Month\ θ	0°	37°	45°	90°
January	52.6°	-15.6°	-7.6°	37.4°
February	44.8°	-7.8°	0.2°	45.2°
March	37°	0°	8°	53°
April	29.1°	17.9°	15.9°	60.9°
May	21.3°	15.7°	23.7°	68.7°
June	13.5°	23.5°	31.5°	76.5°
July	21.3°	15.7°	23.7°	68.7°
August	29.1°	17.9°	15.9°	60.9°
September	37°	0°	8°	53°
October	44.8°	-7.8°	0.2°	45.2°
November	52.6°	-15.6°	-7.6°	37.4°
December	60.5°	-23.5°	-14.5°	29.5°

For maximum solar-energy collection one wants β to be as small as possible, which, for collector angle equal to latitude, θ = φ, would be for the equinox months, March and September. However, it may be desirable to have maximum collection at some other months than the equinox season points; then the angle of the solar collectors θ needs to be some angle different than φ. In the example of φ = 37° in the table above θ = 45° makes the maxima occur for February and October. This collector angle was chosen for the Roper Solar Greenhouse and the author's apartment house so that the maximum would occur for the coldest month of the year, February.

A measure of the cloud cover for each day of a month for Blacksburg VA can be obtained from the NOAA Blacksburg weather station.

The following table lists the collector angles θ for maxima at season points for different northern latitudes (positive angle is toward the equator):

φ:	0°	10°	23.5°	37°	45°	60°	85°	90°
June summer:	-23.5°	-13.5°	0°	13.5°	21.5°	36.5°	61.5°	66.5°
March/September Equinox:	0°	10°	23.5°	37°	45°	60°	85°	90°
December winter:	23.5°	33.5°	47°	60.5°	68.5°	83.5°	-	-

Energy Collected

To calculate the energy collected one needs the following:

- The length of the day for each day of the year at the given latitude.
- Some measure of the cloud cover.
- The angle β of the Sun's rays relative to the perpendicular to the collector.
- The solar energy, insolation, that strikes the surface of the Earth.

Length of Day

The equation for the length of day, d, of year at latitude φ is:

$$d = 24 - \frac{24}{\pi}\arccos\left[\frac{\sin\left(\frac{0.8333\pi}{180}\right)+\sin\left(\frac{\varphi\pi}{180}\right)\sin P}{\cos\left(\frac{\varphi\pi}{180}\right)\cos P}\right]$$

where

$$P = \arcsin\left\{0.39795\cos\left(0.2163108 + 2\arctan\left(0.9671396\tan\left[0.00860(d-186)\right]\right)\right)\right\}.$$

A measure of the cloud cover in tenths of the day length for each day of a month for Blacksburg VA can be obtained from the NOAA Blacksburg weather station. The sum of those numbers is the C used in the equation for the monthly solar energy collected below.

The insolation is ~1000 watts/m^2 at sea level on a clear day.

Example

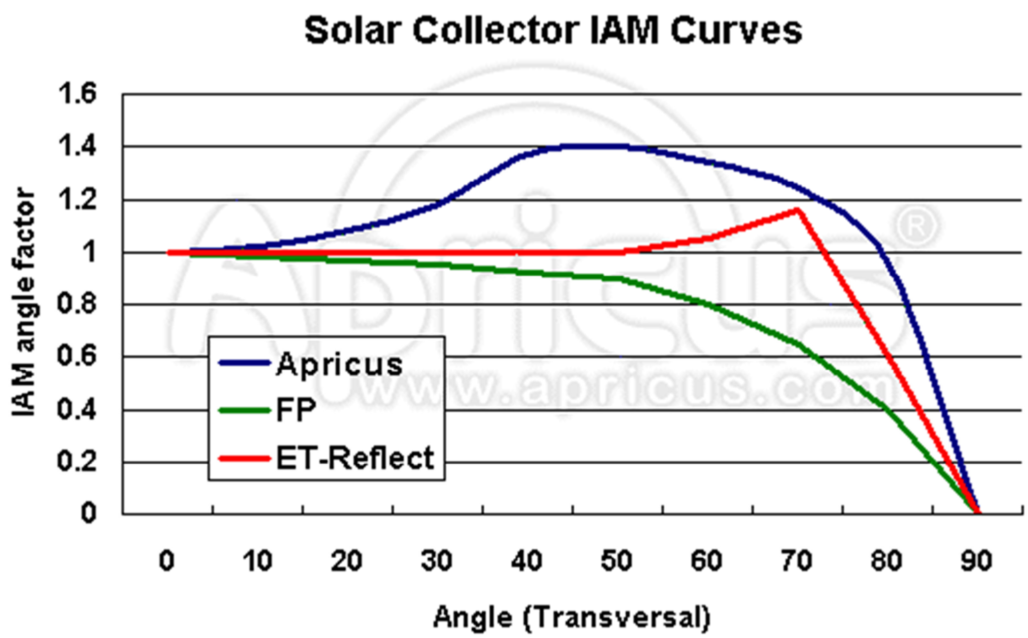
To get an approximate value of the energy (kWh) for each month collected by a particular collector system multiply by 1 kW for each m^2 of the collector system. Actually, the factor should be about I = $\pi/4 \sim 0.785$ kW/m^2, the ratio of the area of a half circle of radius 1 to the area of a rectangle of sides 1 and 2; this accounts for the fact that the Sun's rays east-west transverse movement goes from angle 0 through angle 90º at noon and back to angle 0 when the Sun sets, relative to the plane of the solar collector.

For solar collectors of area 12 m^2 (150 vacuum tubes @ 0.08 m^2 each) and efficiency, e, 0.6 at angle $\theta = A$ at latitude φ = 37º for Blacksburg VA the energy collected each month are:

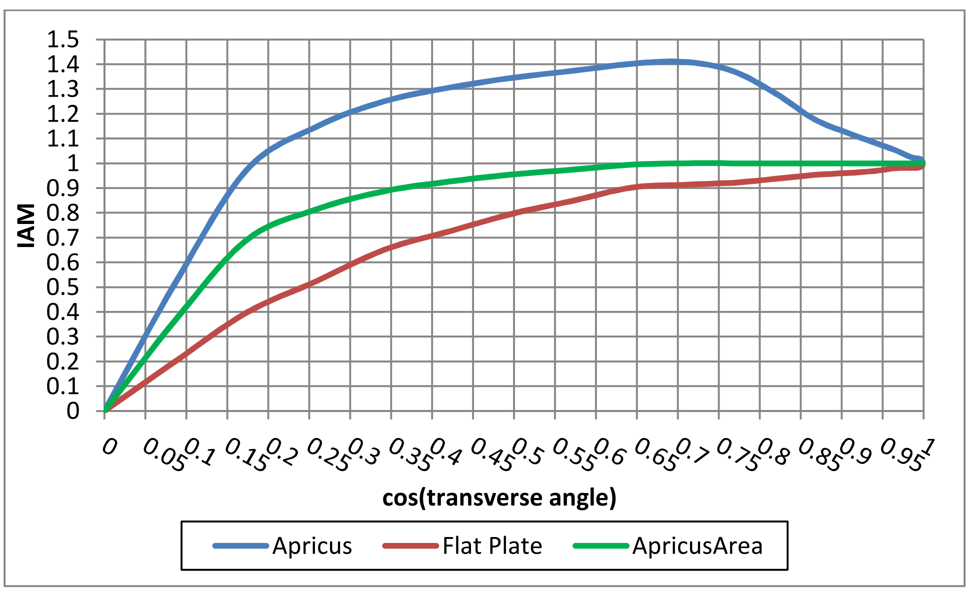
	Day Number		Daylength	Angle	Clouds (tenths)	
Month	J	P	D	A	C	12eID(30-C/10)cosA
Jan-09	15	-0.37	10.17	-8	181	734
Feb-09	45	-0.23	10.84	0	129	926
Mar-09	75	-0.04	11.93	8	146	1096
Apr-09	105	0.17	13.08	16	135	1173
May-09	135	0.33	13.90	23	171	1006
Jun-09	165	0.41	14.25	31	112	1298
Jul-09	195	0.38	14.14	23	144	1222
Aug-09	225	0.26	13.58	16	133	1306
Sep-09	255	0.08	12.59	8	162	973
Oct-09	285	-0.12	11.43	0	150	1034
Nov-09	315	-0.30	10.50	-8	143	923
Dec-09	345	-0.40	10.04	-16	160	819

(For months other than February, April, June, September and November the 30 is replaced by 31 and for February 30 is replaced by 28.)

However, as the following graph shows, the vacuum-tube collectors, e.g. by Apricus, have a higher incidence-angle modifier (IAM) than a standard flat plate collector:



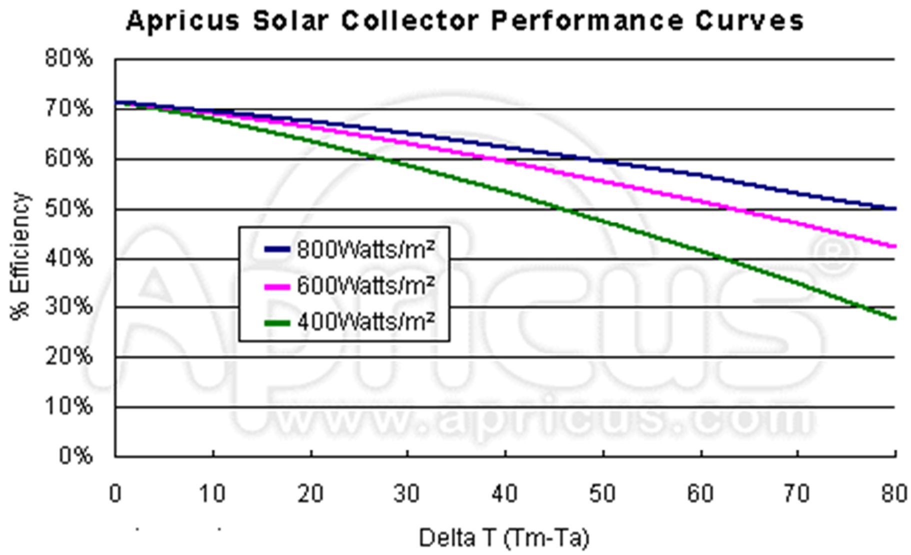
So, vacuum-tube collectors allow a higher collected insolation than flat-plat collectors. However, the Apricus curve above is calculated for the entire area of the Apricus collector system, which includes some empty space between the vacuum tubes. The following graph shows the Apricus and flat-plat IAM curves and a curve for the Apricus system that only includes the actual vacuum-tubes area:



The area under the green curve is 0.881, so 0.881 kW/m^2 instead of 0.785 kW/m^2 for the maximum insolation for the Apricus collectors.

Efficiency

In the calculation above efficiency of 0.6 was used. That was estimated as a reasonable value from the Apricus curves:



I have fitted these curves to the function:

$$Eff = \frac{83.7}{4}\left[1+\tanh\left(\frac{I}{160.68}\right)\right]\left\{1-\tanh\left[\frac{dT-(0.1092I+15.70)}{0.1273I+13.69}\right]\right\},$$

where dT =T_m (Manifold temperature in °C)-T_a (ambient air temperature in °C). As Apricus states "In reality ambient temperature will fluctuate, and the manifold temperature will gradually increase as the water is heated. Furthermore insolation levels may fluctuate with intermittent cloud cover. In order to more accurately calculate energy output per day/month/year a more complete set of environmental data must be considered and many (hourly) performance calculations throughout the day taken."

Apricus gives this equation for the efficiency: Eff = 0.717 – 1.52 dT (1 + 0.0085 dT)/I . This equation is a much worse fit to the data digitized from the three curves than is the more complicated equation. (For example, for I=400 W/m^2 and dT=80°C , this equation gives 20% instead of 28%.) However, when I fit this equation to the three curves I get the best fit to be Eff = 0.7179 – 1.4559 dT (1 + 0.006322 dT)/I . The differences between the values of this equation and the more complicated equation above are not more than 0.6%. The more complicated equation is a better fit to the three curves (chi square of 0.11 compared to 1.77). Using the tanh efficiency equation, the monthly efficiencies vary between 0.623 for Feb 2010 and 0.718 for Jul 2010, instead of the constant 0.6 assumed above.

Energy Collected with Monthly Efficiencies

I want to calculate the average efficiency for each month to include in the calculation of the monthly solar energy collected. To do this I need an good estimate of dT =T_m (Manifold temperature in °C)-T_a (ambient air temperature in °C).

Calculation of dT

In the calculations above for Blacksburg temperature a measure of the cloud cover was obtained from the NOAA Blacksburg weather station. The station also gives the average daily maximum temperature (in daytime) for each month, T_{max}, and the average minimum temperature (in nighttime), T_{min}, for each month. So, a reasonable approximation might be to take the average monthly ambient temperature as

$$T_a = \frac{1}{2}\left[\frac{1}{2}\left(T_{\max} + T_{\min}\right) + T_{\max}\right] = \frac{1}{4}\left(3T_{\max} + T_{\min}\right);$$

i.e., an approximation of the average daytime temperature for the month where $\frac{1}{2}\left(T_{\max} + T_{\min}\right)$ is assumed to be the minimum daytime temperature.

Apricus gives the temperature at which heat dissipators come into play as ~80°C/176°F. So, I assume that the average monthly manifold temperature varies linearly between T_a and 80°C with the energy between 0 and the maximum possible energy collected per month, 12ID31cosθ in the collector. The linear equation is

$$T_m = T_a + \frac{80 - T_a}{12ID31\cos\theta}E,$$

Where E = 12ID(31-C/10)cosθ. So, we have

$$\begin{aligned} dT = T_m - T_a &= \frac{80 - T_a}{12ID31\cos\theta}12ID\left(31 - C/10\right)\cos\theta \\ &= \left(80 - T_a\right)\left(31 - C/10\right)/31 \\ &= \left[80 - \left(3T_{\max} + T_{\min}\right)/4\right]\left(31 - C/10\right)/31 \end{aligned}$$

Of course, April, June, September and November the 31 is replaced by 30and for February 31 is replaced by 28.

Put this equation in for dT in the efficiency equation above.

The calculation done above for constant efficiency of 0.6 now becomes, using the tanh efficiency equation:

	Day Number		Daylength	Angle	Clouds (tenths)		Energy collected/month (kWh)
Month	J	P	D	A	C	Eff	12eID(31-C/10)cosA
Jan-09	15	-0.37	10.17	-8	181	0.641	880
Feb-09	45	-0.23	10.84	0	129	0.651	1127
Mar-09	75	-0.04	11.93	8	146	0.665	1363
Apr-09	105	0.17	13.08	16	135	0.682	1496
May-09	135	0.33	13.90	23	171	0.695	1306
Jun-09	165	0.41	14.25	31	112	0.709	1721
Jul-09	195	0.38	14.14	23	144	0.707	1616
Aug-09	225	0.26	13.58	16	133	0.713	1742
Sep-09	255	0.08	12.59	8	162	0.700	1273
Oct-09	285	-0.12	11.43	0	150	0.683	1320
Nov-09	315	-0.30	10.50	-8	143	0.670	1156
Dec-09	345	-0.40	10.04	-16	160	0.640	979

Possible errors for this calculation:

- The measure of the cloud cover is an approximation of the fraction of the Sun's energy that gets through the clouds.
- The different thicknesses of the atmosphere through which the Sun's rays pass is not accounted for.
- The approximation for dT may not be accurate.